MECH393

Design 2: Machine Element Design

Final Project Report

GROUP 8

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Abstract:

The goal of Solar Impulse, to circumnavigate the world with no fuel, requires a lightweight aircraft. This report outlines the design and optimization process for a gearbox intended for this project. The gearbox must adhere to power and size requirements while optimizing for weight. The gearbox has a double branch double reduction layout, and its components adhere to American Gear Manufacturers Association (AGMA) and American Society of Mechanical Engineers (ASME) standards. All components have a safety factor greater than 1.5, as is industry standard. Detailed analysis and computation lead to a final design weighing 47.38 lbs, outputting a shaft speed of 835 RPM, with a total gear ratio of 6.558. The gearbox is lightweight and is designed to last the full lifetime of 2000 hours.

Statement of Contribution

To Whom it may concern,

This letter states that all this project report on this gearbox design proposal is the collaborative work of all four group members. All four group members directly contributed written parts to this final report, participated in its revision, and contributed CAD parts or worked on the assembly of the final CAD design.

Signed,




Aidan Kimberley Date: December 6, 2024	Antoine Malenfant Date: December 6, 2024	Henri Lapointe Date: December 6, 2024	Laurence Liang Date: December 6, 2024

1. Introduction

The goal of the Solar Impulse project is to fly around the globe with no fuel onboard. This poses many unique challenges, such as optimizing efficiency, weight, and safety. The objective of this project is to design a gearbox for this aircraft. We are supplied with unlimited funds for this project which leads to a high-performance design made to minimize weight. Our gearbox is a double branch double reduction gearbox with the general layout given in Figure 1.

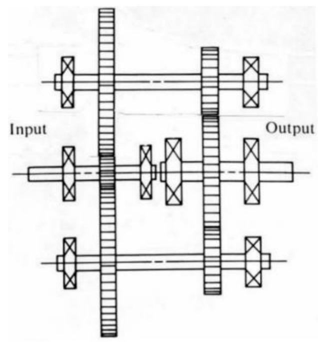


Figure 1: Double Reduction Diagram.

To be conservative we are considering our system to be operating at its maximum power ratings 100% of the time. This means our motor driver, which is connected to the input shaft, will be operating at 60 HP at 5,500 RPM for 2,000 hours. The propeller, which is the output shaft, will be rotating at 835 RPM producing 1,000 lbs. of axial force. Additionally, our gearbox must be less than 30x45x45 cm in dimension. The gearbox specifications are tabulated in Table 1 below.

Table 1: Gearbox Constraints.

Gearbox Specifications	
Max Dimensions (X × Y × Z) [cm]	30 × 45 × 45
Temperature Range [°C]	[-40, 40]
Gear Ratio	6.5882
Power [HP]	60
Safety Factor	1.5
Propeller mass [kg]	100
Thrust [lbs]	1000
Lifetime [h]	2000
Input RPM [rpm]	5500
Output RPM [rpm]	835

Given these constraints, we optimized our design for weight while holding a safety factor of 1.5 due to the high-risk aerospace application. The components of our gearbox are the gears, shafts, keys, and bearings. We iteratively designed these parts following American Gear Manufacturing Association (AGMA) and American Society of Mechanical Engineers (ASME) standards. Figure 2 is a diagram of our gearbox and has the labels we will use to refer to specific parts in this report.

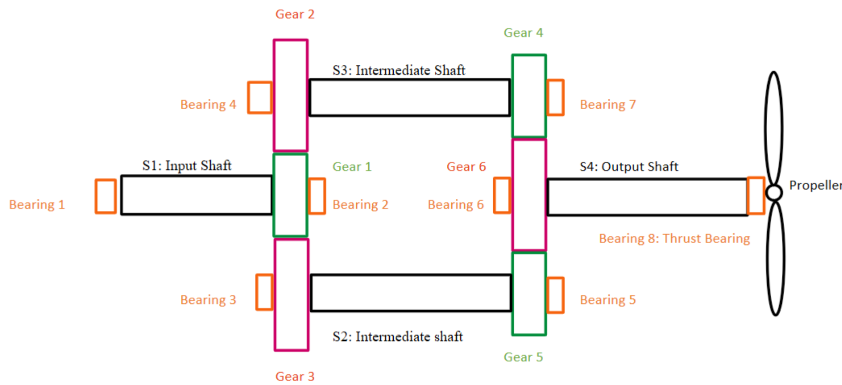


Figure 2: Gearbox Diagram Labelled.

2. Theoretical Development

2.1 Gears

2.1.1 Initial Comments

Note that the gear ratio between gears 2-4 and 3-5 is equal to 1 since they are on the same shaft, i.e. they will have the same rotational velocity. N refers to the number of teeth on a specified gear.

Table 2: Gear Ratio Relationships.

Gear Ratio	Value
Gear Ratio between 1 and 2, m_{21}	N_2/N_1
Gear Ratio between 1 and 3, m_{31}	N_3/N_1
Gear Ratio between 2 and 4, m_{42}	1
Gear Ratio between 3 and 5, m_{53}	1
Gear Ratio between 4 and 6, m_{64}	N_6/N_4
Gear Ratio between 5 and 6, m_{65}	N_6/N_5

All referenced Tables or Figures are in the Annex A and B. The theory behind the analysis process stems from the one described in Chapter 12 of *Machine Design: An Integrated Approach* by Norton. [1]

2.1.2 Design Requirements

Before diving into the design of the gears, requirements need to be specified. As explained in the problem description, the gears will need to output a rotational velocity of 835 rpm when subjected to an input of 60 HP and an input rotational velocity of 5,500 rpm. The maximum lateral distance must be lower than the allowable distance in the wing axis. This value is found in the gear box size requirements and is equal to 45 cm, or 17.7 inches. The maximum lateral distance is given by either of the two following equations, (1) and (2).

$$L_{wing\ axis} = d_{g1} + d_{g2} + d_{g3} + \frac{2}{p_{d1}} \quad (1)$$

$$L_{wing\ axis} = d_{g4} + d_{g5} + d_{g6} + \frac{2}{p_{d2}} \quad (2)$$

Where p_{d1} corresponds to the diametral pitch of gears 1,2 and 3 and p_{d2} corresponds to the diametral pitch of gears 4,5 and 6. d_g corresponds to the pitch diameter of each gear. The allowable distance in the vertical axis is of the same value. Hence, the addendum diameter of the largest gear must not surpass 17.7 inches. As outlined by the project description, the mass of all gears must be minimized as much as possible. In terms of the gear design, this translates by having the lowest total volume. To approximate the total volume of the gears, each gear will be approximated as a cylinder of height equal to the face width of the gear and a cross-sectional area equal to the pitch circle. Consequently, our design will be optimized to have the lowest allowable pitch diameter and face width. Two types of failures will be evaluated for the gears: bending and surface contact (will also be denoted as pitting in this report) failure. Since this project will be completed for an aerospace industry application, the minimal safety factor for both types of failure are required to be equal or above 1.5. Additionally, a contact ratio between 1.4 and 2 is required as this ensures that the load is not concentrated on a singular tooth. It also accounts for errors in tooth spacing that can occur during manufacturing. Finally, the project requires us to use coarse gear, which entails that the diametral pitch of the gears should not surpass 20 (Table 12-2). The following table summarizes these requirements

Table 3: Summary of Design Requirements for Gears.

Parameter	**Requirement**
Length in wing axis	$< 17.7\ in.$
Length in vertical axis	$< 17.7\ in.$
Output RPM, ω_6	$\omega_6 \leq 835\ rpm$
Safety Factor for Bending Failure, N_{fb}	$1.5 \leq N_{fb}$
Safety Factor for Pitting Failure, N_{fc}	$1.5 \leq N_{fc}$
Pitch Diameter, d_p	Minimal
Face Width, F	Minimal
Contact Ratio, m_p	$1.4 < m_p < 2$
Diametral Pitch, p_d	$p_d < 20$

Governing Equations for Gear Design

The equations that governed the design decisions were the safety factor equations, which are functions of the stress and the fatigue strength equations. Those are listed below, in US units.

Bending Stress: $\sigma_b = \frac{W_t p_d K_a K_m K_s K_B K_I}{F J K_v}$ (3)

Bending Fatigue Strength: $S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$ (4)

Surface-contact Stress: $\sigma_c = C_p \sqrt{\frac{W_t C_a C_m C_s C_f}{F I d C_v}}$ (5)

Surface-contact Fatigue Strength: $S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc'}$ (6)

Bending Safety Factor: $N_{fb} = \frac{S_{fb}}{\sigma_b}$ (7)

Surface-contact Safety Factor: $N_{fc} = \frac{S_{fc}}{\sigma_c}$ (8)

As observed from the listed equations, for both bending and pitting, the objective is to minimize the stresses and maximize the strengths.

Constant Parameters

There are quantitative and qualitative parameters that will remain constant for each gear throughout the optimization. First, all gear teeth will have an involute form. This will ensure that center-distance errors in manufacturing and assembly will not affect the velocity ratio. The center-distance between two gears, C, is qualified as the distance between both gear centers. Furthermore, the length of tooth was chosen to be full depth as it will allow more working depth for the gear contact. This is ideal because, for full-depth teeth, the bending geometry factor J, is higher regardless of the pressure angle or the type of loading. As seen in the bending stress equation, there is an inverse proportionality between the bending stress and the bending geometry factor. Additionally, since cost is not an issue for this project, it was assumed that the gears could be precisely manufactured, i.e. manufacturing tolerances will be very small. Hence, the gears' loads will be at the highest point of single-tooth contact (HPSTC). Furthermore, for the cases of full-depth teeth with HPSTC loading, J is higher at a pressure angle of 25˚ compared to 20˚. The pressure angle for each gear mesh contact was decided to be 25˚ to minimize bending stresses as much as possible. As for the quality index of the gears, Q_v, the chosen value is 11. Since our design is for an aircraft engine drive, according to Table 12-6, quality index should be between 10 and 13 [1]. Furthermore, it was initially estimated that the average pitch line velocity for the gears would be in between 2000 and 4000 feet per minute (fpm). A quality index of 11 was then chosen as it was the middle value in the suggested range of gear qualities of Table 12-7. Finally, a reliability of 99 % was chosen as it is considered adequate for aerospace applications.

Table 4: Constant Parameters for Gears.

Parameter	Value
Length of teeth	Full-Depth
Location of Loading	HPSTC
Pressure Angle [degrees]	25
Quality Index	11
Reliability [%]	99

Choice of Material

As explained previously, the objective is to minimize weight while ensuring the gears will not fail in bending or pitting. Therefore, a material with low density and high surface-contact/bending fatigue strengths is required. A selection of materials was presented in the textbook with all the required properties to properly design the gears [1]. The lightest materials to be found were variations of steels, which had strong bending and surface-contact strengths, but not necessarily a low density. By conducting supplementary research outside of the textbook to find a lighter material with similar fatigue strengths, it was quickly realized that the relevant information to assess the validity of our design was most of the time not trustworthy. In other words, there was a lack of confidence in the validity of properties of various materials found outside of the textbook list. Obtaining a complete analysis of our gearbox with a high level of confidence in the properties of the chosen materials was prioritized. Hence, only materials from Table 12-20 and 12-21 were considered [1]. Since all steels had very similar densities, the chosen material for all gears was the highest grade of 2.5% Chrome, Nitrided Steel as it offered the highest bending and surface contact strengths.

Table 5: Chosen Material Properties.

Material Property	Value
Density [lb./in^3]	0.2775
Bending-Fatigue Strength [psi]	65,000
Surface-Contact Strength [psi]	21,6000

Bending Stress Factors

The **Bending Strength Geometry Factor, $\boldsymbol{J}$,** is a function of the number of pinion and gear teeth in a gear mesh. It will only be defined during the optimization process and updated constantly for each iteration. It will be determined using the tabulated values in Table 12-13, which is for full-depth teeth under HPSTC loading at 25° pressure angle [1]. It varies between the pinion and the gear of a singular gear mesh.

The **Dynamic Factor, $\boldsymbol{K_v}$,** accounts for the pitch line velocity, V_t, in fpm and is given by the following equation (9).

$$K_v = \left(\frac{A}{A+\sqrt{V_t}}\right)^B \quad (9)$$

Where $A = 50 + 56(1 - B)$ and $B = \frac{(12-Q_v)^{\frac{2}{3}}}{4}$ for $6 \leq Q_v \leq 11$. Fortunately, our quality index value is 11, which makes this relationship valid. Similarly to the bending strength geometry factor, the value of the dynamic factor will constantly be updated as the number of teeth on the pinion and gear change. The pitch line velocity, V_t, is a function of the rotational speed of the gear, ω, and its pitch diameter, d_p. Pitch diameter will vary depending on the chosen diametral pitch, p_d, i.e. the number of teeth per inch on a gear, and its total number of teeth, N.

$$V_t = \frac{d_p}{2}\omega_p = \frac{d_g}{2}\omega_g \quad (10)$$

$$d_p = \frac{N_p}{p_d};\ \ d_g = \frac{N_g}{p_d} \quad (11)$$

The **Load Distribution Factor, K_m,** is a function of the face width, F. Since this value is minimized as much as possible to have a low weight, it will initially be assumed to be lower than 2 in. This assumption will be confirmed later. Hence, per Table 12-16, the value of K_m will be set equal to 1.6 for each gear [1].

The **Application Factor, K_a,** varies depending on the stability of the driving machine. The motors driving the propeller are electric motors and loads are considered uniformly applied as it is an aircraft with a very niche application, meaning that the motors will be designed as to vibrate and impede as little as possible on the aircraft's performance. Following these assumptions and Table 12-17, the value of K_a will be set equal and constant to 1 for each gear [1].

The **Size Factor, K_S,** is set to 1 since, for this case, there are no situations where the size of a certain geometrical parameter of the gears would affect the overall stress.

The **Rim Thickness Factor, K_B,** accounts for situations where the rim depth is close to the tooth depth. This factor will be revised after the design of the shafts, as they will dictate what each gear bore diameter will have to be. In the case where the rim depth and tooth depth are similar, K_B will be updated using the following equation (12) and will be applied to the concerned gear.

$$K_B = -2\left(\frac{t_R}{h_t}\right) + 3.4 \quad (12)$$

Where t_R is the rim depth and h_t is the tooth depth. For initial conditions, K_B will be set to 1.

The **Idler Factor, K_I,** is set equal to 1 for all gears since there are no idler gears in the design.

Surface-contact Stress Factors

The factors C_a, C_m, C_v and C_s are respectively equal to K_a, K_m, K_v and K_s.

The **Surface Geometry Factor, *I*,** factors in the radii of curvature, ρ_g and ρ_p, and the pressure angle, ϕ. It is identical for both the pinion and the gear. For an external gearset, it is given by the following equations (13) through (15).

$$I = \frac{cos\phi}{\left(\frac{1}{\rho_g} + \frac{1}{\rho_p}\right) d_p} \quad (13)$$

$$\rho_p = \sqrt{\left(r_p + \frac{1 + x_p}{p_d}\right)^2 - \left(r_p cos\phi\right)^2} - \frac{\pi}{p_d} cos\phi \quad (14)$$

$$\rho_g = Csin\phi - \rho_g \quad (15)$$

Where r_p is the pinion pitch radius, x_p is the pinion addendum coefficient, which is equal to 0 for full-depth teeth, C is the center distance between the pinion and the gear. Since some of those values depend on the chosen diametral pitch and number of teeth, this factor will iteratively be updated.

The **Elastic Coefficient, C_P,** accounts for differences in materials. Since all gears are made from the same type of steel, using Table 12-18, C_P is constant and equal to 2300 for each gear [1].

The **Surface Finish Factor, C_F,** is constant and equal to 1 for each gear as there will be no rough surface finishes. As cost is not an issue, proper procedures will be followed to have an ideal surface finish on the gears.

Corrected Fatigue Strengths

For both bending and surface-contact strengths, a correcting factor is known as the life factor and accounts for the expected amount of load cycles of each gear. Before defining each fatigue-strength factor, load cycles for each gear need to be specified. The number of load cycles corresponds to the number of mesh contacts under load of a singular gear tooth. In general, one tooth will experience one load per rotation of the gear. Although, as seen on Figure 2, gears 1 and 6 have two contact meshes. Hence, for one full gear rotation, one tooth will experience two load cycles. Furthermore, the amount of load cycles will also depend on the rotational velocity of the gear. Our input specifies 2000 hours of operation, meaning that gear will rotate at 5500 rpm for 2000 hours. Therefore, gear 1 will complete 6.6×10^8 rotations, which will mean around 1.32×10^9 load cycles. The number of cycles for the other gears ends up being a function of the gear ratio between the other gears. Since they will be rotating at different speeds, they will not complete as many cycles. Although, gears 3 and 5 and 2 and 4 are on the same shaft and will therefore have the same rotational speed and the same amount of load cycles. The following table summarizes the load cycles for each gear.

Table 6: Load Cycles for Each Gear.

Gear Number	**Load Cycles, N_L**

1	$2(6.6 \times 10^8) = 1.32 \times 10^9$
2	$\frac{6.6\times10^8}{m_{21}}$
3	$\frac{6.6\times10^8}{m_{31}}$
4	$\frac{6.6\times10^8}{m_{21}m_{42}}$
5	$\frac{6.6\times10^8}{m_{31}m_{53}}$
6	$\frac{2(6.6\times10^8)}{m_{65}m_{53}m_{31}}$ or $\frac{2(6.6\times10^8)}{m_{64}m_{42}m_{21}}$

Bending-Fatigue Strength Factors

As explained, the **Life Factor, K_L**, accounts for total load cycles of the gear, N_L. The equation used to calculate the factor is given below in equation (16).

$$K_L = 1.3558\, N_L^{-0.0178} \quad (16)$$

This was chosen as it is the upper limit of the range of life factors for loads beyond 10^7 load cycles shown in Figure 9. It will yield a higher bending fatigue-strength, which will allow the design to have a higher bending safety factor. Since most of the total load cycles for each gear is a function of their corresponding gear mesh ratio, this value will update as iterations to optimize the gearbox will be completed [1].

The **Temperature Factor, K_T,** accounts for the operating temperature of the gearbox. According to specifications, that temperature should not surpass 40˚C. Therefore, K_T is set to 1 and remains constant throughout iterations.

The **Reliability Factor, K_R**, is given by Table 12-19, where for a reliability of 99% as precised in the gear assumptions, K_R has a value of 1 and remains constant [1].

Surface-Contact Strength Factors

C_T and C_R are respectively the same as K_T and K_R.

The **Life Factor, C_L,** is calculated in a similar manner as the life factor for bending. From Figure 10, the upper limit of the range of life factors for loads beyond 10^7 load cycles is also taken. Hence, C_L, is given by the following equation (17) [1].

$$C_L = 1.4488\, N^{-0.023} \quad (17)$$

Again, this value will change as change gear dimensions are varied.

The **Hardness Ratio Factor, C_H,** considers the hardness ratio between the materials of the pinion and the gear. Although, for this design, all gears are of the same material. Therefore, this parameter is constant and equal to 1.

Iterative Process

For simplicity, gearset 1 designates gears 1, 2, and 3, and gearset 2 designates gears 4, 5, and 6. After defining all the stress and fatigue-strength factors, whether they were constant or varying, an iterative process was completed to optimize for the lowest gear weight possible. Initially, the face width of each gear was set to its maximal allowed value, which is, according to ASME standard, $16/p_d$. Furthermore, it is ideal to have the same face width, F, and diametral pitch, p_d, within a gear mesh, meaning the gears in gearset 1 will have the same face width and diametral pitch. Gearset 2 will also have the same relationship between its gears, although it is possible that face width and diametral pitch varies from gearset 1 to 2. Another consideration is that the distance between the center of gears 2 and 3 must be the same as the distance between the center gears 4 and 5 to ensure that the gearbox remains symmetrical.

The iterative process was done using the following method: all functions were coded into Excel, which took as inputs the number of teeth on each gear and their assigned diametral pitch. From those inputs, the output rotational velocity, safety factors for bending and surface-contact stresses, total length in the wind axis and center distances between gears 2-3 and gears 4-5 were automatically calculated. If the requirements in terms of output shaft rotational velocity, size and stress safety factors are satisfied, then the chosen geometrical parameters were deemed adequate.

The last parameter that is updated constantly throughout this process that has not been mentioned yet is the tangential force applied on the gear teeth, W_t. It is given by the following equation (18).

$$W_t = \frac{T_p}{r_p} = \frac{T_g}{r_g} \quad (18)$$

Where T_p is the torque applied on the pinion (T_g for gear). As seen in this relationship, the tangential force remains the same for the pinion and the gear of a gear mesh. Furthermore, the torque experienced by a gear will increase if the gear ratio is larger than 1.

The torque at gear 1 is given by the following equation (19).

$$T_1 = \frac{P_{input}}{\omega_1} \quad (19)$$

Since our gearbox requires a rotational speed reduction, gear ratios larger than 1 are expected for each gearset. Hence, the torque is expected to increase between gear 1 and gear 6.

Table 7 summarizes how each geometrical and force parameter will vary for each gear depending on the chosen gear ratio.

Table 7: Rotational Velocity, Torque and Tangential Force for each Gear.

Gear Number	1	2	3	4	5	6
$\boldsymbol{\omega}$	5500	$\frac{5500}{m_{21}}$	$\frac{5500}{m_{31}}$	ω_2	ω_3	$\frac{\omega_2}{m_{64}} = \frac{\omega_3}{m_{65}}$
$\boldsymbol{T}$	T_1	$T_1 * m_{21}$	$T_1 * m_{31}$	T_2	T_3	$T_2 * m_{64} = T_3 * m_{65}$
$\boldsymbol{W_t}$	$\frac{T_1}{r_1}$	$\frac{T_1 * m_{21}}{r_2}$	$\frac{T_1 * m_{31}}{r_3}$	$\frac{T_2}{r_4}$	$\frac{T_3}{r_5}$	$\frac{T_2 * m_{64}}{r_6} = \frac{T_3 * m_{65}}{r_6}$

The first iteration that was done was for the case where the two gearsets had the same gear ratio. To find the individual gear ratio of each gearset, the square root of the overall gear ratio was taken. Using Excel, combinations of gear and pinion teeth numbers were found that matched this ratio. The next step was to choose a diametral pitch. A higher diametral pitch would lead to a smaller gear size and smaller face width. All gear and pinion teeth combinations were tested from the highest possible diametral pitch, i.e. from 18 to a value where the size requirements (17.7 in. in the wing axis) would no longer be met. Unfortunately, for all combinations of gear and pinion teeth, either some gears had safety factors below 1.5 in bending or pitting or were too big to fit in the size constraints. Hence, it was concluded that gearsets 1 and 2 needed different diametral pitches.

The main issue arising from having different diametral pitches between gearset 1 and 2 is that the symmetry of the gearbox might not be respected. In other words, the center distances between gears 2-3 and gears 4-5 need to be equal. This was done by applying the following constraint on the number of teeth on gear 6.

$$d_1 + d_2 = d_4 + d_6 \quad (20)$$

$$\frac{(N_1 + N_2)}{p_{d_1}} = \frac{(N_4 + N_6)}{p_{d_2}} \quad (21)$$

$$N_6 = \frac{p_{d_2}}{p_{d_1}}(N_1 + N_2) - N_4 \quad (22)$$

The result is rounded to the closest integer. p_{d_1} and p_{d_2} correspond to the pitch diameter of gearsets 1 and 2 respectively. Since gearset 1 will have higher rotational speeds, it was assumed that it would have a higher diametral pitch. Furthermore, the torque is higher on gearset 2, it would then have a lower diametral pitch. This would lead to bigger teeth on the gear which would help sustain higher stresses. Although there is a tradeoff, since for a higher pitch diameter a higher number of teeth might be required to respect the required safety factor, which might make the gears too big.

The iteration process for this part went as follows:

- Set pitch diameter of gear 1 to highest possible value, i.e. 18.
- Adjust gear 1 teeth number to ensure that bending and surface-contact safety factors are satisfied.
- Adjust gears 2-3 number of teeth to have a gear ratio, m_{21} and m_{31}, close to 2.566 (value determined previously). Ensure that total length in wing axis is lower than 17.7 in.
- Set pitch diameter of gears 4-5 to highest possible value, i.e. 18.
- Adjust gears 4-5 number of teeth to ensure that overall gear ratio satisfies output rotational velocity. Verify if bending and surface-contact safety factors are satisfied.
- Gear 6 number of teeth is automatically updated through constraint in equation (22).

The following actions were taken depending on each case described:

- If bending and surface-contact safety factors are not satisfied for gears 4-5, reduce diametral pitch to closest standard value from Table 12-2.
- If, for any diametral pitch value for gears 4-5, output rotational velocity of gearbox is not adequate, increase number of teeth on gears 2-3 until maximum wing axis length is achieved. If output velocity is still not respected when that length is reached, reduce gear 1 diametral pitch to next closest value in Table 12-2.
- Repeat process until all requirements are satisfied.

For each valid combination of diametral pitch and number of teeth, the face width was reduced as much as possible without compromising the safety factors to minimize mass. After iterating through that process, the lightest combination of diametral pitch and gear teeth that respected requirements was chosen. It is described in the results part of this report. The stress and strength factors used to calculate the safety factors for that chosen design are listed in Table 8 and Table 9 below.

Table 8: Bending and Surface-Contact Stress Factors.

Gear Number	**1**	**2**	**3**	**4**	**5**	**6**
Bending						
J	0.47	0.512	0.512	0.4025	0.4025	0.4875
K_v	0.8765	0.8765	0.8765	0.9152	0.9152	0.9152
K_m	1.6	1.6	1.6	1.6	1.6	1.6
K_a	1	1	1	1	1	1
K_S	1	1	1	1	1	1
K_B	1	1	1	1.548	1.548	1
K_I	1	1	1	1	1	1
Surface-Contact						
C_a	1	1	1	1	1	1
C_m	1.6	1.6	1.6	1.6	1.6	1.6
C_v	0.8765	0.8765	0.8765	0.9152	0.9152	0.9152

C_s	1	1	1	1	1	1
C_P	2300	2300	2300	2300	2300	2300
I	0.1365	0.1365	0.1365	0.1109	0.1109	0.1109
C_F	1	1	1	1	1	1

Table 9: Fatigue-Strength Factors.

Gear Number	1	2	3	4	5	6
Bending						
K_L	0.9329	0.9590	0.9590	0.9590	0.9590	0.9648
K_T	1	1	1	1	1	1
K_R	1	1	1	1	1	1
Surface-Contact						
C_L	0.8938	0.9262	0.9262	0.9262	0.9262	0.9334
C_T	1	1	1	1	1	1
C_R	1	1	1	1	1	1
C_H	1	1	1	1	1	1

As explained previously, gears 4 and 5 have a similar rim depth compared to their tooth depth. Hence, the rim thickness factor was adjusted.

A sample calculation for safety factor of gear 4 is provided in Annex C.

2.2 Shafts

2.2.1 Preliminary Design

Before proceeding with a preliminary design of the shafts, some assumptions were made for the entirety of the shaft assembly. First, the material remained the same throughout the entirety of the design process. SAE 1020 Machined Steel of $S_{ut} = 65\ kpsi$ and $S_y = 38\ kpsi$ [1] was used uniformly across the shafts, as it provides a relatively light metal with a reasonable strength, which makes it useful for simple shaft assemblies and general machinery. Furthermore, considering that the weight of the shaft is predicted to be significantly lower than that of the gear, the different materials were not explored to ensure a more efficient and concise design process. Second, a notch radius of 0.01 inch was assumed throughout all stress concentrations. From this assumption, notch sensitivity in bending and torsion could be calculated from the notch radius and the ultimate strength of the steel. Note that for torsion, a curve of 20 kpsi greater was used to calculate the notch sensitivity factor. From there, another assumption was made concerning the stress concentration factor. It was assumed to be 3.5 in bending, 2 in torsion, and 4 at the keys. These assumptions were made in agreement with Peterson's *Stress Concentration Factors,* which display these values as approximate maxima for the expected loadings of the shafts assembly [2]. Hence, these factors will be used all throughout, yielding results with a factor of safety slightly higher than reality. All parameters related to stress concentrations can be found in Table 10 below.

Table 10: Design Parameters for Stress Concentrations.

Parameter Name	Parameter Symbol	Value [unitless]
Notch radius	r	0.01
Stress Concentration Factor in Bending	K_t	3.5
Stress Concentration Factor in Torsion	K_{ts}	2.0
Stress Concentration Factor at Keyhole	$K_{t,key}$	4.0

From these stress concentration factors, the fatigue factors were calculated in each case and were used throughout this section to calculate the diameters of the shafts. From the calculations, it was determined that in all cases, the fatigue concentration factors were equal to their mean counterpart. This was done by verifying that the maximum nominal stress did not increase past the ratio of yield strength over fatigue concentration factor. These values can hence be found in Table 11 below.

Table 11: Design Parameters for Fatigue Factors.

Parameter Name	Parameter Symbol	Value [unitless]
Notch Sensitivity in Bending	$q_{bending}$	0.5
Notch Sensitivity in Torsion	$q_{torsion}$	0.57
Fatigue Concentration Factor in Bending	K_f	2.25
Fatigue Concentration Factor in Torsion	K_{fs}	1.57
Mean Fatigue Concentration Factor in Bending	K_{fm}	2.25
Mean Fatigue Concentration Factor in Torsion	K_{fsm}	1.57
Fatigue Concentration Factor in Bending at Keyhole	$K_{f,key}$	2.5
Fatigue Concentration Factor in Torsion at Keyhole	$K_{fs,key}$	2.7
Mean Fatigue Concentration Factor in Bending at Keyhole	$K_{fm,key}$	2.5
Mean Fatigue Concentration Factor in Torsion at Keyhole	$K_{fsm,key}$	2.7

From the material selection, the required strengths could be obtained. As the endurance strength of the material is of interest for the design of the shaft, it was calculated using equation (23).$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e{}' S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e{}'$

$$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e{}' \quad (23)$$

The uncorrected endurance strength, $S_e{}'$, was calculated from the steel standards recommending a value of half the ultimate tensile strength for steels with an ultimate tensile strength superior to 200 kpsi, as suggested in Figure 7. The coefficients could then be obtained. See Table 12 for the coefficient values for the corrected endurance strength.

$\boldsymbol{C_{load}}$ is linked to the loading effects. In our case, almost exclusively bending and torsion are present. However, a thrust load is present on the output shaft. It was determined that the effects of the thrust force had negligible effect on the diameter calculations compared to the output torque, and hence it was not considered in the calculations of the load coefficient. Given that for both bending and torsion the respective load coefficients are equal to 1, it was set accordingly.

$\boldsymbol{C_{size}}$ is liked to size effects. From Figure 8, the size coefficient is equal to 1 for diameters larger than 0.3 inch, which is expected in our case.

$\boldsymbol{C_{surf}}$ is liked to surface effects. Our shaft material is expected to be machined and will be calculated using the following equation (24). From Figure 8, the coefficients for machined material can be found.

$$C_{surf} = A(S_{ut})^b \qquad (24)$$

$\boldsymbol{C_{temp}}$ is liked to temperature effects. It is assumed that the assembly is used at room temperature, and hence the coefficient is equal to 1.

$\boldsymbol{C_{reliab}}$ is linked to the reliability of the material. We chose our material to have a reliability of 99.9999%, hence the coefficient can be found to be 0.62 from Table 40.

Table 12: Design Parameters for Endurance Strength.

Parameter Name	Parameter Symbol	Value	Units
Uncorrected Endurance Strength	$S_e{}'$	32500	psi
Load Coefficient	C_{load}	1	-
Size Coefficient	C_{size}	1	-
Surface Coefficient	C_{surf}	0.84	-
Temperature Coefficient	C_{temp}	1	-
Reliability Coefficient	C_{reliab}	0.62	-
Endurance Strength	S_e	16926	psi

Before entering the design phase for each of the shafts, an overall dimensioning of the shaft assembly was made as a preliminary constraint. The maximum length was set to a maximum of 30 cm, or 11.81 inches in our case. The overall goal was to not go over 11 inches and hence remain within the desired dimensions. The assumed dimensions in the following sections were hence decided from this constraint.

It should be noted that the lengths of every shaft potion supporting the bearings were left as variables in the design process. This was done as these sections support only bearing reaction forces and will later be optimized to fit the final assembly. The exact dimensions will be found in Table 34 in the result section.

Note that the equations used in this sections come from Norton, Machine Design: An Integrated approach unless specified [1].

2.2.1.1 Input Shaft

From the project definition, the preliminary parameters were defined. It was outlined that the input shaft would have a maximum RPM of 5500, along with a maximum horsepower of 60HP [3]. Throughout the design of this gearbox, it was assumed that maximum loading conditions were present. From these design parameters, the torque applied to the input shaft was calculated. For reference in the design process, a rough schematic of the input shaft is included in Figure 3 below.

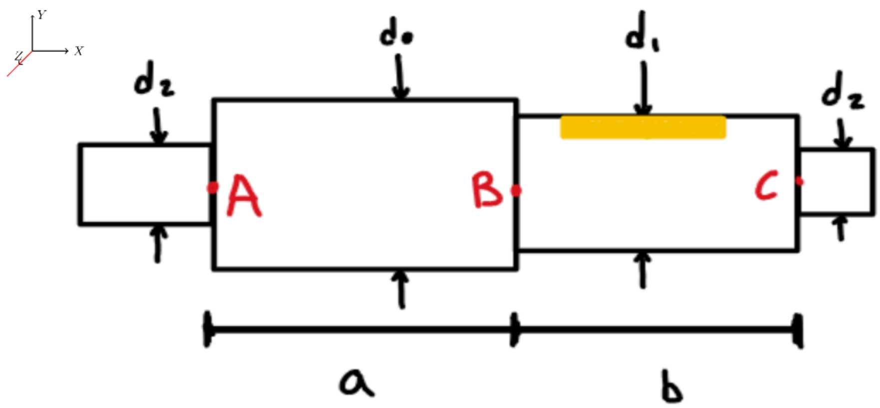


Figure 3: Input Shaft Schematic.

From this initial idea of a design, bearing reaction forces along with moments induced by the gears could be calculated. To achieve this, dimensions needed to be set. From the length constraint set in the previous subsection, values of length were set to a = 2 in and b = 1 in. Hence, reaction forces and moments at various points can be found in Table 14 below. Note that points A, B, and C from Figure 3: Input Shaft Schematic. Figure 3 are used as references. Note also that bearings (B1 and B2) will be situated at points A and C, and a gear 1 will be situated at point B along with its respective key.

Table 13: Dimensions for the Input Shaft.

Dimension	Value [in]
a	2
b	1

From the load configuration, one may observe that no tangential or radial force is acting on the shaft because of the symmetric gears, which will result in no forces and/or moments being induced on the shafts caused by the gears. Hence, the preliminary design of the input shaft was initially solely dependent on the torque applied. However, upon consideration for the overall gear assembly weight, it was decided that the weight of the gears would be considered as an alternating load on the shaft. Hence, alternating moments were calculated all throughout the shaft. Note that the reaction forces will be displayed in the sample calculation in the annex for clearer display of results. The relevant loading parameters for the design of the input shaft can be found in Table 14 below.

Table 14: Loading Parameters for the Input Shaft.

Parameter Name	Parameter Symbol	Value	Units
Mean Torque	T_m	687.5493542	Lb-in
Moment at point A	M_A	0	Lb-in
Moment at point B	M_B	1.5762	Lb-in
Moment at point C	M_C	0	Lb-in

Then, using the equation for diameter displayed below in equation (25), the preliminary shaft minimum diameters at different points of interest could be calculated. It should be noted that a factor of safety of 1.5 was used to obtain these results. The initial minimum diameters can be found in Table 15 below. Note that the equation used is equation 10.8 by Norton in Machine Design: An Integrated Approach [1]. It should also be noted that fatigue factors for keys were used for the calculations of d_1, as a gear and key combination will be present at this location.

$$d = \left\{ \frac{32N_f}{\pi} \left[\frac{\sqrt{(K_f M_a)^2 + \frac{3}{4}(K_{fs} T_a)^2}}{S_e} + \frac{\sqrt{(K_{fm} M_m)^2 + \frac{3}{4}(K_{fsm} T_m)^2}}{S_{ut}} \right] \right\}^{\frac{1}{3}} \quad (25)$$

Table 15: Initial Diameters Values for Input Shaft.

Diameter	Value [in]
d_0	0.606360916
d_1	0.725239877
d_2	0.603444579

2.2.1.2 Intermediate Shafts

The intermediate shafts design process was slightly different, as tangential and radial forces from the gears were now considered. This hence meant that larger reaction forces and, inevitably, alternating moments would be present. As per the input shaft, the first step done was to calculate the torque acting on the shaft. Once again, this torque would be acting as a mean torque in our calculations. In this case, the power transmission through gears is done using the transmitted force. Hence, given that these forces are the ratio between the torque and gear radius, the transmitted torque could easily be calculated using the following equation (26).

$$T_{gear} = \frac{T_{pinion} * r_{gear}}{r_{pinion}} \quad (26)$$

Next, the dimensioning of the shaft was decided. To optimize the design, they were based on the face width of the gears, and the *a* dimension was decided based to minimize the gap between B2 and B6. See Figure 2 for reference. A schematic of one intermediate shaft can be seen in Figure 4 below, along with Table 16 displaying the dimensions.

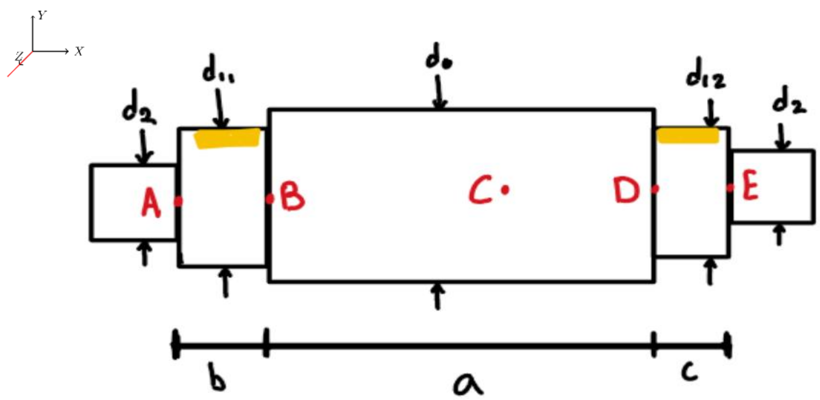


Figure 4: Intermediate Shaft Schematic.

Table 16: Dimensions for the Intermediate Shaft.

Dimension	Value [in]
a	3
b	0.75
c	2

Taking one singular shaft, one can observe that gear forces will be present at both point B and D, see Figure 4 above. Hence, this will induce reaction forces at point A and E, meaning at the bearing's location. Note that the direction of the gear forces will be opposite for the two intermediary shafts but will be equal in scalar values as the gears are designed to be symmetrical. They will hence transmit equivalent radial (z-direction) and tangential (y-direction) forces. However, the tangential values will not be equal because of the gear weight configuration. Hence, the moments and forces will have to be calculated for both shafts to ensure a safe design. However, the final shafts will be designed to be identical and satisfy both load configurations. See Table 17 below for the different load parameters. Once again, only the torque and moments will be displayed, and the relevant forces were calculated as displayed in sample calculations found in the annex. Note also that the total moments are the sum of moments in the y and z direction, given that gear forces have both a radial and tangential component and hence induce moments in both directions.

Table 17: Loading Parameters for the Intermediate Shaft.

Parameter Name	Parameter Symbol	Value [lb-in]
Mean Torque	T_m	1617.763186
Total Moment at point A	M_A	0

Total Moment at point B	M_B	113.2760408
Total Moment at point C (Max value)	M_C	1807.541477
Total Moment at point D	M_D	1807.541477
Total Moment at point E	M_E	0

**Note that the value of moment at C is the maximum between values at B and D to consider the max loading in the section.*

Then, using equation (25) used in the input shaft, the minimum diameter of the shaft at the various locations were calculated. Once again, a factor of safety of 1.5 was used for the calculations. The initial minimum diameters can be found in Table 18 below.

Table 18: Initial Diameters Values for Intermediate Shaft.

Diameter	**Value [in]**
d_0	1.542656639
d_{11}	1.046110219
d_{12}	1.706352725
d_2	0.802615878

2.2.1.3 Output Shaft

The design of the output shaft resembled the process of the input shaft, as it is between two identical pinions, transmitting equal and opposite forces to the gear. Hence, the shaft does not have any tangential transmitted load that induces a bending moment. However, the weight of the gear is considered and hence induces a moment. The gear present on the output shaft (G6) is the heaviest one in the gearset at 20.11 lbs and is the main reason why the effect of weight was considered. As per equation (26) used for the intermediate shafts, the transmitted torque was calculated. Furthermore, the dimensions of the output shaft were also determined. Note that the dimensions were determined based on the Figure 10-5 on page 601 of Norton [1]. The dimensiond_3 was calculated apart of the design problem, but if can be changed depending on the fitting of the propeller. A schematic of the output shaft is seen in Figure 5 below, along with the dimensions of the shaft in Table 19.

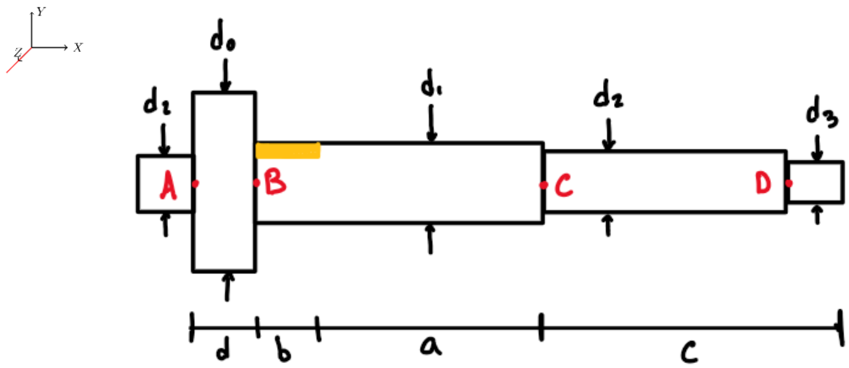


Figure 5: Output Shaft Schematic.

Table 19: Dimensions for the Output Shaft.

Dimension	Value [in]
a	1.5
b	2
c	3.9370
d	1

In this configuration, the weight of the gear will have a downward force at B, while the weight of the propeller will create a downward force at D. These forces will hence induce bending moments throughout the shafts. Note that those forces, along with the reaction forces from the bearings will be displayed in the appendix.

Unlike the other shafts of this gearbox, this output shaft is subjected to an axial force, which needs to be considered for the design of this shaft. However, this axial force does not induce any moment and hence equation (25) will not be used. The loading parameters can be seen in Table 20 below.

Table 20: Loading Parameters for the Output Shaft.

Parameter Name	Parameter Symbol	Value	Units
Mean Torque	T_m	4529.736922	Lb-in
Moment at point A	M_A	0	Lb-in
Moment at point B	M_B	177.3165882	Lb-in
Moment at point C	M_C	868.3014219	Lb-in
Moment at point D	M_D	0	Lb-in
Axial Thrust	T_{axial}	1000	Lb

To calculate the diameter of the shaft, the basic equation of line CD of the Goodman diagram was used. For reference, see Figure 6 in the appendix below.

The equation (25) was derived from the intersection of line CD for a constant ratio of alternating and mean stresses. However, it was derived under the assumption that axial loads were 0 and only torsional and bending loadings were present. Upon derivations, equation (27) below was obtained.

$$N_f = \left\{ \frac{32K_f M_a}{S_e \pi d^3} + \frac{\sqrt{\left(\frac{4K_{fm}T_{axial}}{\pi d^2}\right)^2 + 3\left(\frac{16K_{fsm}T_m}{\pi d^3}\right)^2}}{S_{ut}} \right\}^{-1} \quad (27)$$

To find values for the shaft diameters, an iterative method using excel was conducted as the difference in power of the diameter values in the equation makes it hard to solve. This formula was then iterated with steady-increasing values of diameter until the output factor of safety reached a value of 1.5. This was done using the different values of alternating moments and fatigue concentration factors for each location of interest. The obtained values of diameters can be found in Table 21 below.

Table 21: Initial Diameters Values for Output Shaft.

Diameter	Value [in]
d_0	1.47590
d_1	1.42474
d_2	1.47585
d_3	1.21871

2.2.2 Corrected Design

Upon the completion of the preliminary design, some changes were made. It was brought to our attention that the formulas used assumed a failure due to fatigue, and hence the factor of safety represented a fatigue factor of safety. However, it meant that the calculated diameters may have the required factor of safety in failure, but not in yielding. Hence, the obtained diameters were used to calculate the factors of safety, and needed changes were calculated. The diameters were put into the equation of line CD and DE, seen in equations (28) and (29) respectively.

$$\frac{\sigma'_m}{S_{ut}} + \frac{\sigma'_a}{S_e} = \frac{1}{N_f} \quad (28)$$

$$\frac{\sigma'_m}{S_y} + \frac{\sigma'_a}{S_y} = \frac{1}{N_y} \quad (29)$$

2.2.2.1 Input Shafts

The initial diameters of the input shaft along with the calculated fatigue and yielding factors of safety can be found in Table 22 below. From equation (25), coming from (28) and (29), the following equations were derived.

$$N_f = \frac{\pi d^3}{32}\left[\frac{\sqrt{\left(K_f M_a\right)^2 + \frac{3}{4}\left(K_{fs} T_a\right)^2}}{S_e} + \frac{\sqrt{\left(K_{fm} M_m\right)^2 + \frac{3}{4}\left(K_{fsm} T_m\right)^2}}{S_{ut}}\right]^{-1} \quad (30)$$

$$N_y = \frac{\pi d^3}{32}\left[\frac{\sqrt{\left(K_f M_a\right)^2 + \frac{3}{4}\left(K_{fs} T_a\right)^2}}{S_y} + \frac{\sqrt{\left(K_{fm} M_m\right)^2 + \frac{3}{4}\left(K_{fsm} T_m\right)^2}}{S_y}\right]^{-1} \quad (31)$$

Table 22: Initial Diameters of Input Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	0.606360916	1.5	0.886336176
d_1	0.725239877	1.5	0.883012937
d_2	0.603444579	1.5	0.876923077

From Table 22, all the shafts would yield before failing from fatigue. Hence, the initial diameters that were calculated did not meet the requirements for our gearbox. From there, new diameters were calculated from a refined formula of diameter seen in equation (32). Through these calculations, a yielding factor of safety of 1.5 was used. This equation is based on equation (28) and (29) above. The new diameters along with their factors of safety can be seen in Table 23 below.

$$d = \left\{\frac{32N_y}{\pi}\left[\frac{\sqrt{\left(K_f M_a\right)^2 + \frac{3}{4}\left(K_{fs} T_a\right)^2}}{S_y} + \frac{\sqrt{\left(K_{fm} M_m\right)^2 + \frac{3}{4}\left(K_{fsm} T_m\right)^2}}{S_y}\right]\right\}^{\frac{1}{3}} \quad (32)$$

Table 23: Corrected Diameters of Input Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	0.722595847	2.538540184	1.5
d_1	0.865345899	3.402796457	1.5
d_2	0.721684388	2.565789474	1.5

With these corrected minimum values, final shaft values were determined. Given that the governing dimension is d_2 since it needs to be the size of a bearing bore, it was determined first. It was scaled up to the closest bearing bore diameter. From this increase, the other dimensions were increased accordingly to maintain the proportions of the dimensions and to fit the initial design idea as seen in Figure 3. The value of d_0 was hence increased as to create a notch for better gear-shaft assembly. The final dimensions can be found in Table 24 below.

Table 24: Final Diameters of Input Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1	6.728180883	3.975620078
d_1	0.9	2.866643986	1.687522483
d_2	0.7974	3.461049275	2.023382653

2.2.2.2 Intermediate Shafts

The initial diameters of the intermediate shaft along with the calculated fatigue and yielding factors of safety can be found in Table 25 below.

Table 25: Initial Diameters of Intermediate Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1.542656639	1.5	2.491004521
d_{11}	1.046110219	1.5	1.036551228
d_{12}	1.706352725	1.5	2.230123343
d_2	0.802615878	1.5	0.876923077

Hence, the values of d_{11} and d_2 do not satisfy the minimum factor of safety of 1.5 in yielding. These values were recalculated using equation (32) above. The corrected minimum diameters can hence be found in Table 26 below.

Table 26: Corrected Diameters of Intermediate Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1.542656639	1.5	2.491004521
d_{11}	1.046110219	1.5	1.036551228
d_{12}	1.706352725	1.5	2.230123343
d_2	0.802615878	1.5	0.876923077

In a similar manner as for the input shaft, new dimensioning was set based on the closest bearing bore diameter from d_2 going up. The final values can be found in Table 27 below along with both factors of safety.

Table 27: Final Diameters of Input Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1.8	2.096304622	3.481269528
d_{11}	1.2	2.308087871	1.585494234
d_{12}	1.71	1.509639182	2.247030921

d_2	0.984251969	2.766221132	1.617175431

2.2.2.3 Output Shafts

The initial diameters of the output shaft along with the calculated fatigue and yielding factors of safety can be found in Table 28 below.

Table 28: Initial Diameters of Output Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1.21871	1.5	1.028232947
d_1	1.42474	1.5	0.976896916
d_2	1.47585	1.5	1.475719678
d_3	1.13176	1.5	1.094737076

It can be observed from Table 28 that all calculated diameters would fail in yielding before failing in fatigue. The iterative process was redone using equation (33) below. Note that equation is a combination of equation (27) and (29) above.

$$N_f = \left\{ \frac{32K_f M_a}{S_y \pi d^3} + \frac{\sqrt{\left(\frac{4K_{fm}T_{axial}}{\pi d^2}\right)^2 + 3\left(\frac{16K_{fsm}T_m}{\pi d^3}\right)^2}}{S_y} \right\}^{-1} \quad (33)$$

The iterative process was done until both factors of safety were ensured to be larger than the required minimum of 1.5. The corrected diameters and their factors of safety can be seen in Table 29 below.

Table 29: Corrected Diameters of Output Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1.38238	2.188367228	1.5
d_1	1.6439	2.303580794	1.5
d_2	1.48391	1.524714676	1.5
d_3	1.3538	2.565932146	1.5

In a similar manner as for the input and intermediate shaft, new dimensioning was set based on the closest thrust bearing bore diameter from d_2 going up. The final values can be found in Table 30 below along with both factors of safety. Note that as per the input shaft, the value of d_0 was increased as to create a notch for better gear-shaft assembly.

Table 30: Final Diameters of Input Shaft and Factors of Safety.

Diameter	Value [in]	N_f	N_y
d_0	1.7	4.066597775	2.787034707
d_1	1.65	2.329302164	1.516939676
d_2	1.57480315	1.822166948	1.792473313
d_3	1.36	2.601301992	1.520761165

2.3 Bearings

We design our bearings based on these three design criteria:

- Load rating: we verify that the static load rating is higher than the static applied loads on the bearing.
- Maximum operating speed: we verify that the bearing's maximum expected rotational speed is less than the maximum designed rotational speed for the bearing
- Number of loading cycles: we want to ensure that the L_P (number of cycles that the bearing can withstand before 1% of the bearings fail, a failure rate that we defined which we'll explain more in detail below)

Since the gearbox plays a critical role in transmitting power from the motor to the propeller, we chose conservative design criteria in order to increase the reliability of the bearings. As a result, we introduced a safety factor of SF=1.5: this choice of value is a common safety factor in aerospace, allows for a reasonable amount of margin between our selected bearings' properties and the anticipated loads and usage, and is small enough such that the safety factor doesn't require the bearings to be overly large or heavy.

Load Rating Criterion

For the load rating design criteria, the following relation determines whether radial bearing's load rating can handle load reactions

$$C_0 > SF \times P \qquad (34)$$

where C_0 is the static load rating for our bearing, SF=1.5 is the safety factor, and P is the magnitude of the applied loads on that bearing. For radial loading, following Norton's example, we calculate the magnitude of applied loads P as

$$P = \sqrt{F_y^2 + F_z^2} \qquad (35)$$

In the case of combined axial and radial loading, we use the following relation to calculate P:

$$P = X \cdot V \cdot F_r + Y \cdot F_a \qquad (36)$$

where V, X, Y are coefficients that we obtain from Fig. 11.24 from Norton based on the ratio of the axial load and the radial load, and the static load rating of the bearing. To keep this current section concise, we included a detailed calculation in the appendix with a numerical example to determine the specific values for each variable.

Maximum Operating Speed Criterion

Given the maximum operating speed desired for each shaft and the bearing's maximum operating speed, we use the following relation to determine whether a bearing satisfies the maximum operating speed criterion:

$$\omega_{\max bearing\ speed} > SF \times \omega_{\max desired} \qquad (37)$$

where SF=1.5 is the safety factor.

Load Cycles Criterion

Given the equivalent applied load P and the bearing's dynamic load rating C, we use the following relation to determine the number of maximum cycles that the bearing can withstand up to 10% of its components failing (the L_{10} number). Furthermore, because all of our bearings are roller bearings, we set the exponent term to 10/3:

$$L_{10} = \left(\frac{C}{P}\right)^{\frac{10}{3}} \qquad (38)$$

Next, we add the reliability factor K_R to calculate the number of cycles for a smaller failure rate. As we want to maximize bearing reliability, especially for the gearbox which plays a critical role in the airplane, we want a reliability of 99% and a failure rate of 1%. We apply this same reliability factor to all L_{10} calculations, which is K_R=0.21 for a 1% failure rate as defined by Table 11-5 from Norton (included in the appendix):

$$L_1 = K_{R=0.99} \times L_{10} \qquad (39)$$

Finally, we want to verify that the resulting L_1 value is greater than the maximum anticipated number of cycles, assuming a worst case scenario where the shaft continuously experiences the maximum rotational speed:

$$L_1 > SF \times L_{max\ cycles} \qquad (40)$$

where SF=1.5 is the safety factor and $L_{max\ cycles}$ is the max number of cycles that the shaft would experience at the highest possible rotational speed.

Bearing Selection Process

We also determined that the diameter of each shaft acts as a constraint for the bore diameter of each bearing. Accordingly, we determined that the bearing's bore diameter must be larger than the minimum shaft diameter estimate, but ideally smaller than the adjacent shaft notch's diameter. The bore diameter's lower bound ensures that the shaft diameter is sufficiently large to withstand support reactions for its designed operating lifespan, while the bore diameter's upper bound should allow the bearing's overall diameter to be smaller than its adjacent gears. While there are no hard upper bounds for the bore diameter since we could machine additional notches, we do not want the bore diameter to be too large. An oversized bore diameter means that the shaft diameter is overdesigned at that site, and the bearings would take up an exceedingly large amount of volume in the gearbox, making the design sub-optimal. Additionally, an oversized bore diameter would require machining additional notches on the shaft, which adds additional complexity for manufacturing the shaft and for analyzing stress concentrations.

As a result, for each bearing under radial loading conditions, we started with the first bearing whose bore diameter was above the minimum shaft diameter. We quickly realized that the ball bearings provided in Norton wouldn't necessarily last the number of load cycles given our design criteria and the applied loads on our shaft. As a result, we first turned to the NSK Ltd bearings catalogue, as these set of bearings had high load ratings and had a high variety of options that we could choose from. [4] However, we realized that the NSK catalogue did not offer an appropriate thrust bearing that was sufficiently small to match our desired bore diameter of 40mm that could support combined axial and radial applied loads. As a result, we decided to switch all our bearings to the SKF catalogue [5], as we found a tapered roller bearing thrust bearing that had our desired bore diameter of 40mm and still withstand our loading conditions. We made sure that the thrust bearing could support both radial and axial loads, as some thrust bearings only support axial loading. After we solved the thrust bearing design bottleneck, we were quickly able to find candidate bearings that only needed to support radial loading for all the remaining bearing sites.

In an idealized setting, we would first start with the bearing with the smallest bore diameter, run the load cycle calculations based on the bearing's load ratings, and iterate with progressively larger bearings until we satisfy all our design criteria. However, in our case, the choice of bore diameter would also affect the shaft design: since the shaft is arguably more critical (as there are more stringent design requirements for the shaft subject to our failure analyses), we would first determine the minimum shaft diameter, determine an ideal largest bore diameter based on the adjacent notch diameters on the shaft, and use these two values as minimum and maximum thresholds for our bore diameter. As a result, we would start with the smallest available bearing within this range, and progressively iterate with bearings with higher load ratings if needed. At bearing sites 1 and 2 where the applied loads are very small, the number of safe loading cycles would be several orders of magnitude larger than the target number of loading cycles. We decided to keep these bearings regardless of this apparent difference in magnitude since these were the smallest bearings available based on our design criteria, and the cost of these bearings is manageable given our project budget. In general, we decided to use roller bearings instead of ball bearings, because roller bearings can support higher rotational speeds and applied loading than equivalent ball bearings.

In some cases that experienced high loading conditions, we added some extra margins on top of our safety margins: as we were continuously iterating our shaft and gear designs in parallel, we wanted to add some extra margins for the bearing requirements to ensure that changes in the shaft design could fluctuate within a reasonable amount and still stay within all of our safety requirements for the bearing design criteria.

2.4 Keys

We needed keys in our design to transmit torque from the shafts to the gears as well as hold them in place. We used a parallel key design to assess the stresses in our gearbox to simplify our calculations, however, in practice we would use a tapered key to prevent axial slipping of our gears. The general dimensions of the tapered key will be the same as the parallel keys. The ASME standards define standard key sized for shaft diameters, shown in Table 10 from Norton. [1] Therefore, the only design variables were the material and the length of the key. We chose a low carbon ASE 1010 steel because it is weaker than the shafts and gears and we want the keys to fail before the more expensive parts. However, we still want to maintain our minimum safety factor of 1.5 due to our high risk aerospace application and our large budget.

Keys fail in two ways: bearing and shear. Shear failure is fatigue failure due to the shearing of the key between the gear and the shaft. To evaluate failure we must find the von mises stress. In our application we have constant torque with no alternating component so the safety factor can be calculated as follows:

$$F = \frac{Torque}{r},\ \tau = \frac{F}{width*length},\ \sigma' = \sqrt{3}\tau,\ N_{shear} = \frac{s_{ut}}{\sigma'} \quad (41)$$

Bearing failure is from the compressive stress due to the contact between the key and the shaft. Bearing stress is compressive therefore we consider it static. We can calculate the safety factors with the following equations:

$$\sigma_{bearing} = \frac{Force}{\frac{1}{2}*height*length},\ N_{bearing} = \frac{s_y}{\sigma_{bearing}} \quad (42)$$

We designed the lengths to set the safety factor of all of our keys greater than 1.5 while constraining the keyway length to less than 1.5 times the diameter of the shaft to prevent excessive twisting and shaft deflection.

3. Results

3.1 Gears

Below are listed the parameters and calculated safety factors of our final gear designs for each gear. The following tables include proof of requirement satisfaction as well as the weights of the gears.

Table 31: Gear Geometrical Parameters and Safety Factors.

Gear Number	Diametral pitch, p_d [tooth/in.]	Number of teeth, N	Face Width, F [in.]	Bore Diameter , D_b [in.]	Pitch Diameter, d_p [in.]	Pitch Radius, r_p [in.]	Bending Safety Factor, N_{fb}	Pitting Safety Factor, N_{fc}
1	12	34	0.75	0.9	2.83	1.415	2.19	1.52
2	12	80	0.75	1.2	6.67	3.335	2.07	2.41
3	12	80	0.75	1.2	6.67	3.335	2.07	2.41
4	8	20	2	1.71	2.5	1.25	2.17	1.51
5	8	20	2	1.71	2.5	1.25	2.17	1.51
6	8	56	2	1.65	7	3.5	2.79	2.55

Table 32: Proof of Requirement Satisfaction for Gears.

Material	High Grade, 2.5 % Chrome, Nitrided Steel
Maximal Length in wing axis [in.]	16.337
Length in vertical axis [in.]	7.25
Output RPM	834.8
Minimum Safety Factor for Bending Failure	2.07
Minimum Safety Factor for Pitting Failure	1.51
Pitch Diameter	Within values for coarse gears
Contact Ratio for Gearset 1	1.534
Contact Ratio for Gearset 2	1.479
Center distance between gears 2 and 3 [in.]	9.5
Center distance between gears 4 and 5 [in.]	9.5

Table 33: Gear Volume and Weights.

Gear Number	Volume [in.^{3}]	Weight [lbs.]
1	4.26	1.182
2	24.52	6.804
3	24.52	6.804
4	5.42	1.504
5	5.42	1.504
6	72.46	20.11
Total	136.6	37.91

3.2 Shafts

Table 34: Shafts Diameter Final Values.

Location	Diameter	Value [in]	Nom. Length [in]	N_f	N_y
Input shaft	d_{B1}	0.7974	1.00	3.461049275	2.023382653
	$d_{0.Input}$	1.0000	2.00	6.728180883	3.975620078
	d_{G1}	0.9000	1.00	2.866643986	1.687522483
	d_{B2}	0.7974	0.65	3.461049275	2.023382653
Intermediate Shaft	d_{B3-B4}	0.9842	1.00	2.766221132	1.617175431
	d_{G2-G3}	1.2000	0.75	2.308087871	1.585494234
	$d_{0.Intermediate}$	1.8000	3.00	2.096304622	3.481269528
	d_{G4-G5}	1.7100	2.00	1.509639182	2.247030921
	d_{B5-B7}	0.9842	1.00	2.766221132	1.617175431
Output Shaft	d_{B6}	1.5748	1.00	1.822166948	1.792473313
	d_{G6}	1.6500	3.50	2.329302164	1.516939676
	$d_{0.Output}$	1.7000	1.00	4.066597775	2.787034707
	d_{B8}	1.5748	3.94	1.822166948	1.792473313
	d_{prop}	1.3600	0.50	2.601301992	1.520761165

3.3 Bearings

Table 35: Bearing Names and General Specifications. Bearings from SKF [4]

Bearing Number	Bearing Name	Bore Diameter [in]	Bearing Type	Weight (lbs.)
B1	SKF N204 ECP	0.787	Single row cylindrical roller bearing	0.2381
B2	SKF N204 ECP	0.787	Single row cylindrical roller bearing	0.2381
B3	SKF NU1005	0.984	Single row cylindrical roller bearing	0.183
B4	SKF NU1005	0.984	Single row cylindrical roller bearing	0.183
B5	SKF NJ 2305 ECML	0.984	Single row cylindrical roller bearing	0.8598
B6	SKF NU1008 ML	1.575	Single row cylindrical roller bearing	0.4894
B7	SKF NJ 2305 ECML	0.984	Single row cylindrical roller bearing	0. 8598
B8	SKF 33208	1.575	Single row tapered roller bearing	1.583

Table 36: Desired Design Requirements and the Corresponding Bearing Properties.

Desired Design Requirements and the Corresponding Bearing Properties						
	Number of Cycles		Applied Loading		Rotational Speed	
Bearing	Desired number of cycles	Calculated L_1	P (lb)	C_0 (lb)	Desired RPM	Max Rated RPM
B1	6.60E+08	2.269E+18	0.788	4,946	5,500	19,000
B2	6.60E+08	2.287E+19	0.394	4,946	5,500	19,000

B3	2.806E+08	9.960E+09	126.3	2,967	2,338	18,000
B4	2.806E+08	1.224E+10	118.7	2,967	2,338	18,000
B5	2.806E+08	3.292E+09	793.3	12,364	2,338	22,000
B6	1.002E+08	3.275E+10	177.3	5,845	835	18,000
B7	2.806E+08	3.343E+09	789.6	12,364	2,338	22,000
B8	1.002E+08	1.245E+09	2,124.1	29,675	835	8,500

3.4 Keys

After performing the calculations in Section in 2.4, we determined the dimensions for our keyways and keys that satisfy our constraints.

Table 37: Key Dimensions.

Key #	**Dimensions (l x w x h) [in]**			$N_{bearing}$	N_{shear}
1	0.5	0.25	0.25	1.80	2.50
2	0.75	0.25	0.25	1.53	2.12
3	0.75	0.25	0.25	1.53	2.12
4	0.5	0.375	0.375	2.18	3.03
5	0.5	0.375	0.375	2.18	3.03
6	1.25	0.375	0.375	1.88	2.61

3.5 Gearbox Dimensions and Weight

Total Weight	47.38 lbs
Gearbox Dimensions	16.7 x 11.2 x 7.25 [in]
Final RPM	834.82 RPM
Gear Ratio	6.5881

The technical drawings of the gearbox assembly are located in the Appendix, Section 6.

4. Conclusion

Through an iterative design process, our group designed a gearbox that weights 47.38 lbs., has a dimension of 16.7 x 11.2 x 7.25 in., has a final output rotational speed of 834.82 RPM with a gear ratio of 6.5881, which is suitable to use as a gearbox for the Solar Impulse airplane's electric population system. By assuming a continuous, worst-case max loading scenario, we tested all the components of our design accordingly for a total operating time of 2,000 hours subject to all relevant failure conditions and assumed a high degree of reliability. As a result, we were able to satisfy all design criteria (including the gearbox dimensions, operating life of 2,000 hours), matched the rotational output speed within 0.18 RPM of the target 835 RPM (which we judge to be sufficiently close), and minimized the weight to 47.38 lbs., making our gearbox a desirable proposed design for the Solar Impulse airplane. Eventually, other materials for the parts with high strength-to-weight ratio such as composites and other strong metals such as titanium could be considered. Provided that the necessary properties are available in trustworthy literature, the weight could be minimized even more.

References

[1] R. L. Norton, Machine Design: An Integrated Approach, 6th edition, Pearson, 2020.

[2] R. Peterson, Stress Concentration Factors, John Wiley, 1974.

[3] M. 393, Project Description - Design Project, 2024.

[4] SKF, "Rolling bearings," 2024. [Online]. Available: https://www.skf.com/sg/products/rolling-bearings.

[5] NSK, "Rolling Bearings for Industrial Machinery," 2016. [Online]. Available: https://www.nsk.com/content/dam/nsk/common/catalogs/ctrgPdf/bearings/e1103b.pdf.

5. Annex

Annex A: Figures.

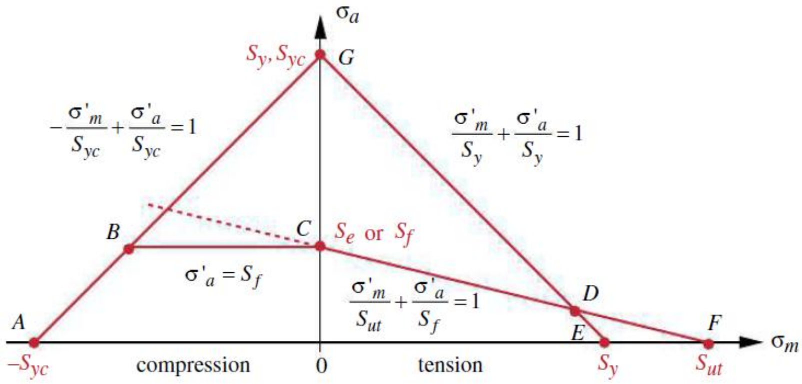


Figure 6: An "Augmented" Modified-Goodman Diagram for an Even Material (Norton, 2020).

$$\text{steels:} \quad \begin{cases} S_{e'} \cong 0.5\,S_{ut} & \text{for } S_{ut} < 200 \text{ kpsi } (1\,400 \text{ MPa}) \\ S_{e'} \cong 100 \text{ kpsi } (700 \text{ MPa}) & \text{for } S_{ut} \geq 200 \text{ kpsi } (1\,400 \text{ MPa}) \end{cases}$$

$$\text{irons:} \quad \begin{cases} S_{e'} \cong 0.4\,S_{ut} & \text{for } S_{ut} < 60 \text{ kpsi } (400 \text{ MPa}) \\ S_{e'} \cong 24 \text{ kpsi } (160 \text{ MPa}) & \text{for } S_{ut} \geq 60 \text{ kpsi } (400 \text{ MPa}) \end{cases}$$

$$\text{aluminums:} \quad \begin{cases} S_{f'_{@5E8}} \cong 0.4\,S_{ut} & \text{for } S_{ut} < 48 \text{ kpsi } (330 \text{ MPa}) \\ S_{f'_{@5E8}} \cong 19 \text{ kpsi } (130 \text{ MPa}) & \text{for } S_{ut} \geq 48 \text{ kpsi } (330 \text{ MPa}) \end{cases}$$

$$\text{copper alloys:} \quad \begin{cases} S_{f'_{@5E8}} \cong 0.4\,S_{ut} & \text{for } S_{ut} < 40 \text{ kpsi } (280 \text{ MPa}) \\ S_{f'_{@5E8}} \cong 14 \text{ kpsi } (100 \text{ MPa}) & \text{for } S_{ut} \geq 40 \text{ kpsi } (280 \text{ MPa}) \end{cases}$$

Figure 7: Uncorrected Endurance Limits for Various Materials [1].

$$\begin{aligned} &\text{for } d \leq 0.3 \text{ in } (8 \text{ mm}): && C_{size} = 1 \\ &\text{for } 0.3 \text{ in} < d \leq 10 \text{ in}: && C_{size} = 0.869 d^{-0.097} \\ &\text{for } 8 \text{ mm} < d \leq 250 \text{ mm}: && C_{size} = 1.189 d^{-0.097} \end{aligned}$$

Figure 8: Size Factors Calculations Guidelines [1].

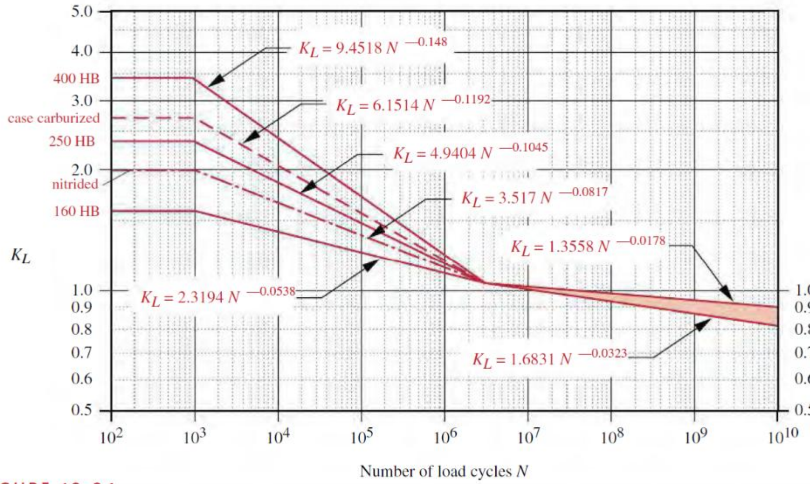


FIGURE 12-24

Bending Strength Life Factor K_L Source: Extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth* with the permission of the publisher, American Gear Manufacturers Association, 1001 N. Fairfax St., Suite 500, Alexandria, VA 22314.

Figure 9:Bending Strength Life Factor, K_L[1].

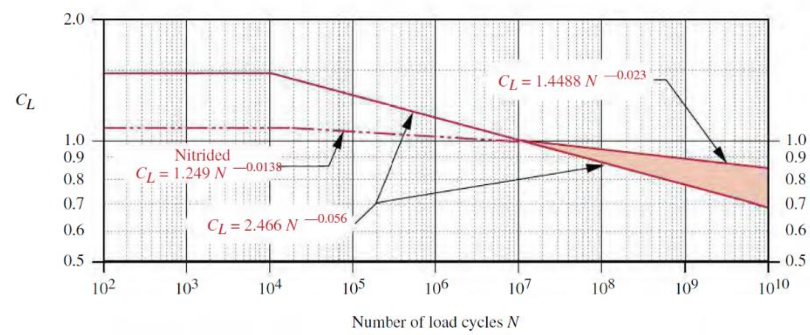


FIGURE 12-26

AGMA Surface-Fatigue Strength Life Factor C_L Source: Extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth* with the permission of the publisher, American Gear Manufacturers Association, 1001 N. Fairfax St., Suite 500, Alexandria, VA 22314.

Figure 10: Surface-Fatigue Strength Life Factor, C_L[1].

Factors V, X, and Y for Radial Bearings

Bearing Type 3)	4)	5)	In Relation to the Load the Inner Ring is: Rotating	Stationary	Single Row Bearings 1) $\frac{F_a}{VF_r} > e$		Double Row Bearings 2) $\frac{F_a}{VF_r} \leq e$		$\frac{F_a}{VF_r} > e$		e
	$\frac{F_a}{C_0}$	$\frac{F_a}{iZD_w^2}$	V	V	X	Y	X	Y	X	Y	
Radial Contact Groove Ball Bearings	0.014	25				2.30				2.30	0.19
	0.028	50				1.99				1.99	0.22
	0.056	100				1.71				1.71	0.26
	0.084	150				1.55				1.55	0.28
	0.11	200	1	1.2	0.56	1.45	1	0	0.56	1.45	0.30
	0.17	300				1.31				1.31	0.34
	0.28	500				1.15				1.15	0.38
	0.42	750				1.04				1.04	0.42
	0.56	1000				1.00				1.00	0.44
20°					0.43	1.00		1.09	0.70	1.63	0.57
25°					0.41	0.87		0.92	0.67	1.44	0.68
30°			1	1.2	0.39	0.76	1	0.78	0.63	1.24	0.80
35°					0.37	0.66		0.66	0.60	1.07	0.95
40°					0.35	0.57		0.55	0.57	0.93	1.14
Self-Aligning Ball Bearings			1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 tan α
Self-Aligning and Tapered Roller Bearings			1	1.2	0.40	0.4 cot α	1	0.45 cot α	0.67	0.67 cot α	1.5 tan α

1) For single row bearings, when $\frac{F_a}{VF_r} \leq e$ use $X = 1$ and $Y = 0$.

For two single row angular contact ball or roller bearings mounted "face-to-face" or "back-to-back" the values of X and Y which apply to double row bearings. For two or more single row bearings mounted "in tandem" use the values of X and Y which apply to single row bearings.

2) Double row bearings are presumed to be symmetrical.

3) Permissible maximum value of $\frac{F_a}{C_0}$ depends on the bearing design.

4) C_0 is the basic static load rating.

5) Units are pounds and inches.

Values of X, Y and e for a load or contact angle other than shown in the table are obtained by linear interpolation.

FIGURE 11-24

V, X, and *Y* Factors for Radial Bearings *Excerpted with permission from SKF roller bearings catalogue, 2012. Copyright SKF Group 2012.*

ANNEX B: Tables

Table 38:Nominal Key Widths for Various Shaft Diameters [1].

Shaft Diameter (in)	Nominal Key Width (in)
$0.312 < d \leq 0.437$	0.093
$0.437 < d \leq 0.562$	0.125
$0.562 < d \leq 0.875$	0.187
$0.875 < d \leq 1.250$	0.250
$1.250 < d \leq 1.375$	0.312
$1.375 < d \leq 1.750$	0.375
$1.750 < d \leq 2.250$	0.500
$2.250 < d \leq 2.750$	0.625

Table 39: Coefficients for Surface-Factor [1].

Table 6-3 Coefficients for Surface-Factor Equation 6.7e

Some data taken from Shigley, Mischke and Budynas, *Mechanical Engineering Design*, 7th ed., McGraw-Hill, New York, 2004, p. 329

	For S_{ut} in MPa, use		For S_{ut} in psi, use	
Surface Finish	*A*	*b*	*A*	*b*
Ground	1.58	–0.085	2.411	–0.085
Machined or cold-rolled	4.51	–0.265	16.841	–0.265
Hot-rolled	57.7	–0.718	2052.9	–0.718
As-forged	272	–0.995	38 545.0	–0.995

Table 40: Reliability Factors [1].

Reliability %	C_{reliab}
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620

Table 41: Reliability Factors for a Weibull Distribution.

Reliability Factors *R* for a Weibull Distribution Corresponding to the Probability of Failure *P*

P%	R%	K_R
50	50	5.0
10	90	1.0
5	95	0.62
4	96	0.53
3	97	0.44
2	98	0.33
1	99	0.21

Gear Tables

Table 12-2
Standard Diametral Pitches

Coarse ($p_d > 20$)	Fine ($p_d \geq 20$)
1	20
1.25	24
1.5	32
1.75	48
2	64
2.5	72
3	80
4	96
5	120
6	
8	
10	
12	
14	
16	
18	

Table 12-6
Suggested Gear Quality Numbers for Various Applications

Application	Q_v
Cement mixer	3–5
Cement kiln	5–6
Steel mill drives	5–6
Cranes	5–7
Punch press	5–7
Conveyor	5–7
Packaging machinery	6–8
Power drill	7–9
Washing machine	8–10
Printing press	9–11
Automotive transmission	10–11
Marine transmission	10–12
Aircraft engine drive	10–13
Gyroscope	12–14

Table 12-7
Suggested Gear Quality Numbers versus Pitch Line Velocity

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

Table 12-13 AGMA Bending Geometry Factor J for 25°, Full-Depth Teeth with HPSTC Loading

	Pinion teeth															
Gear teeth	12		14		17		21		26		35		55		135	
	P	G	P	G	P	G	P	G	P	G	P	G	P	G	P	G
12	U	U														
14	U	U	0.33	0.33												
17	U	U	0.33	0.36	0.36	0.36										
21	U	U	0.33	0.39	0.36	0.39	0.39	0.39								
26	U	U	0.33	0.41	0.37	0.42	0.40	0.42	0.43	0.43						
35	U	U	0.34	0.44	0.37	0.45	0.40	0.45	0.43	0.46	0.46	0.46				
55	U	U	0.34	0.47	0.38	0.48	0.41	0.49	0.44	0.49	0.47	0.50	0.51	0.51		
135	U	U	0.35	0.51	0.38	0.52	0.42	0.53	0.45	0.53	0.48	0.54	0.53	0.56	0.57	0.57

Table 12-16

Load Distribution Factors K_m

Face Width in (mm)	K_m
<2 (50)	1.6
6 (150)	1.7
9 (250)	1.8
≥20 (500)	2.0

Table 12-17 Application Factors K_a

Driving Machine	Driven Machine: Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light Shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium Shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

Table 12-18 Elastic Coefficient C_p in Units of $[psi]^{0.5}$ ($[MPa]^{0.5}$)†

Pinion Material	E_p psi (MPa)	Gear Material: Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30*E*6 (2*E*5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable Iron	25*E*6 (1.7*E*5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular Iron	24*E*6 (1.7*E*5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast Iron	22*E*6 (1.5*E*5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum Bronze	17.5*E*6 (1.2*E*5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin Bronze	16*E*6 (1.1*E*5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

†The values of E_p in this table are approximate and ν = 0.3 was used as an approximation of Poisson's ratio for all materials. If more accurate numbers are available for E_p and ν, they should be used in equation 11.23 to obtain C_p.

Table 12-19

Reliability Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Table 12-20 Bending-Fatigue Strengths S_{fb}' for a Selection of Gear Materials*†

Material	Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Bending-Fatigue Strength psi x 10^3	Bending-Fatigue Strength MPa
Steel	A1—A5		Through hardened	≤180 HB	25—33	170—230
			Through hardened	240 HB	31—41	210—280
			Through hardened	300 HB	36—47	250—325
			Through hardened	360 HB	40—52	280—360
			Through hardened	400 HB	42—56	290—390
			Flame or induction hardened	Type A pattern 50—54 HRC	45—55	310—380
			Flame or induction hardened	Type B pattern	22	150
			Carburized and case hardened	55—64 HRC	55—75	380—520
		AISI 4140	Nitrided	84.6 HR15N	34—40	230—310
		AISI 4340	Nitrided	83.5 HR15N	36—47	250—325
		Nitralloy 135M	Nitrided	90.0 HR15N	38—48	260—330
		Nitralloy N	Nitrided	90.0 HR15N	40—50	280—345
		2.5% Chrome	Nitrided	87.5—90.0 15N	55—65	380—450
Cast iron	20	Class 20	As cast		5	34
	30	Class 30	As cast	175 HB	8.5	59
	40	Class 40	As cast	200 HB	13	90
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	22—33	152—228
	A-7-c	80-55-06	Quenched and tempered	179 HB	22—33	152—228
	A-7-d	100-70-03	Quenched and tempered	229 HB	27—40	186—276
	A-7-e	120-90-02	Quenched and tempered	269 HB	31—44	213—303
Malleable iron (pearlitic)	A-8-c	45007		165 HB	10	70
	A-8-e	50005		180 HB	13	90
	A-8-f	53007		195 HB	16	110
	A-8-i	80002		240 HB	21	145
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	5.7	40
	Al/Br 3	ASTM B-148 alloy 954	Heat treated	90 ksi min tensile strength	23.6	160

*Some data extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 N. Fairfax St., Suite 500, Alexandria, VA 22314.
† Rockwell 15N scale used for case-hardened materials see Section 2-4

Table 12-21 Surface-Fatigue Strengths S_{fc}' for a Selection of Gear Materials*†

Material	Class	Material Designation	Heat Treatment	Minimum Surface Hardness	Surface-Fatigue Strength psi x 10^3	Surface-Fatigue Strength MPa
Steel	A1–A5		Through hardened	≤ 180 HB	85–95	590–660
			Through hardened	240 HB	105–115	720–790
			Through hardened	300 HB	120–135	830–930
			Through hardened	360 HB	145–160	1000–1100
			Through hardened	400 HB	155–170	1100–1200
			Flame or induction hardened	50 HRC	170–190	1200–1300
			Flame or induction hardened	54 HRC	175–195	1200–1300
			Carburized and case hardened	55–64 HRC	180–225	1250–1300
		AISI 4140	Nitrided	84.6 HR15N†	155–180	1100–1250
		AISI 4340	Nitrided	83.5 HR15N	150–175	1050–1200
		Nitralloy 135M	Nitrided	90.0 HR15N	170–195	1170–1350
		Nitralloy N	Nitrided	90.0 HR15N	195–205	1340–1410
		2.5% Chrome	Nitrided	87.5 HR15N	155–172	1100–1200
		2.5% Chrome	Nitrided	90.0 HR15N	192–216	1300–1500
Cast iron	20	Class 20	As cast		50–60	340–410
	30	Class 30	As cast	175 HB	65–75	450–520
	40	Class 40	As cast	200 HB	75–85	520–590
Nodular (ductile) iron	A-7-a	60-40-18	Annealed	140 HB	77–92	530–630
	A-7-c	80-55-06	Quenched and tempered	180 HB	77–92	530–630
	A-7-d	100-70-03	Quenched and tempered	230 HB	92–112	630–770
	A-7-e	120-90-02	Quenched and tempered	230 HB	103–126	710–870
Malleable iron (pearlitic)	A-8-c	45007		165 HB	72	500
	A-8-e	50005		180 HB	78	540
	A-8-f	53007		195 HB	83	570
	A-8-i	80002		240 HB	94	650
Bronze	Bronze 2	AGMA 2C	Sand cast	40 ksi min tensile strength	30	450
	Al/Br 3	ASTM B-148 78 alloy 954	Heat–treated	90 ksi min tensile strength	65	450

*Some data extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 N. Fairfax St., Suite 500, Alexandria, VA 22314.
† Rockwell 15N scale used for case-hardened materials—see Section 2-4

Annex C: Gear Sample Calculation

As specified, a sample calculation for the safety factors of gear 4 is shown.

Geometrical Parameters

$$diametral\ pitch, p_d = 8$$

$$pinion\ number\ of\ teeth, N_4 = 20$$

$$gear\ number\ of\ teeth, N_6 = 56$$

$$Initial\ face\ width, F = \frac{16}{p_d} = 2\ in.$$

$$Pitch\ diameter, d_p = \frac{N_4}{p_d} = \frac{20}{8} = 2.5\ in.$$

$$Pitch\ radius, r_p = \frac{d_p}{2} = 1.25\ in.$$

$$Center\ distance\ between\ gears\ 4\ and\ 6, C = 4.75\ in.$$

$Clearance, c = \frac{0.157}{p_d}$. This parameter is added for manufacturing purposes.

$$Bore\ diameter, D_b = 1.71\ in.$$

To determine tooth depth, the addendum and dedendum diameters are required.

$$Addendum\ Diameter, D_a = d_p + \frac{2}{p_d} = 2.5 + \frac{2}{8} = 2.75\ in.$$

$$Dedendum\ Diameter, D_d = d_p - 2\left(\frac{1}{p_d} + c\right) = d_p - 2\left(\frac{1}{p_d} + \frac{0.157}{p_d}\right) = 2.5 - 2\left(\frac{1}{8} + \frac{0.157}{p_d}\right)$$

$$D_d = 2.21\ in.$$

$$Tooth\ depth, h_t = (D_a - D_d)/2 = 0.27\ in.$$

$$Rim\ depth, t_r = (D_d - D_b)/2 = 0.25\ in.$$

$$addendum\ coefficient, x_p = 0$$

Input Torque

$$T_1 = \frac{P_{input}}{\omega_1} = \frac{\left(60hp\left(6600\frac{\frac{in-lb}{s}}{hp}\right)\right)}{5500\ rpm\left(\frac{2\pi}{60}\right)\frac{rad}{s}/rpm} = 687.6\ lbs$$

Rotational velocity of gear 4

$$\omega_4 = \frac{\omega_1}{m_{21}m_{42}} = \frac{(5500\ rpm)}{\frac{N_2}{N_1}1} = \frac{5500}{\frac{80}{34}} = 2337.5\ rpm$$

Tangential velocity of gear 4

$$V_t = \omega_4 r_p = (2337.5\ rpm)(1.25\ in.)(2\pi)(1ft/12\ in.) = 1529.9\ fpm$$

Tangential force of gear 4

$$W_{t_4} = \frac{T_4}{r_4} = \frac{T_2}{r_2} = \frac{T_1 * m_{21}}{r_p} = \frac{687.6\left(\frac{80}{34}\right)}{1.25} = 1296.4\ lbs$$

Stress factors

As explained previously, $K_m, K_a, K_s, K_I, C_m, C_a, C_s, C_p, C_F$ were already determined using the assumptions described in section 2.1 of this report.

- Bending Strength Factor, *J*

  J is determined by interpolating the values in table 12-13 for $N_p = 20$ and $N_g = 54$.

$$J = 0.4025$$

- Surface Geometry Factor, *I*

$$\rho_p = \sqrt{\left(r_p + \frac{1 + x_p}{p_d}\right)^2 - \left(r_p \cos\phi\right)^2} - \frac{\pi}{p_d}\cos\phi$$

$$\rho_p = \sqrt{\left(1.25 + \frac{1+0}{8}\right)^2 - (1.25\cos 25)^2} - \frac{\pi}{8}\cos 25 = 0.423$$

$$\rho_g = Csin\phi - \rho_g$$

$$\rho_g = 4.75sin25 - 0.423 = 1.584$$

$$I = \frac{cos25}{\left(\frac{1}{1.584} + \frac{1}{0.423}\right) 2.5} = 0.1109$$

- Dynamic Factor, K_v, C_v
  Note Q_v=11;

$$K_v = C_v = \left(\frac{A}{A + \sqrt{V_t}}\right)^B$$

$$B = \frac{(12 - Q_v)^{\frac{2}{3}}}{4} = \frac{(12 - 11)^{\frac{2}{3}}}{4} = 0.25$$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.25) = 92$$

$$K_v = C_v = \left(\frac{A}{A + \sqrt{V_t}}\right)^B = \left(\frac{92}{92 + \sqrt{1529.9\ fpm}}\right)^{0.25} = 0.9152$$

- Rim-Thickness Factor, K_B

$$K_B = -2\left(\frac{t_R}{h_t}\right) + 3.4 = -2\left(\frac{0.25\ in.}{0.27\ in.}\right) + 3.4 = 1.548$$

Bending and Surface-Contact Stresses

$$Bending\ stress, \sigma_b = \frac{W_t p_d K_a K_m K_s K_B K_I}{FJK_v}$$

$$\sigma_b = \frac{(1296.4)(8)(1)(1.6)(1)(1.548)(1)}{(2)(0.4025)(0.9152)} = 34{,}976.84\ psi$$

$$Surface - contact\ stress, \sigma_c = C_p \sqrt{\frac{W_t C_a C_m C_s C_f}{FIdC_v}}$$

$$\sigma_c = (2300)\sqrt{\frac{(1296.4)(1)(1.6)(1)(1)}{(2)(0.1109)(2.5)(0.9152)}} = 147{,}026.091\ psi.$$

Strength factors

As explained previously, $K_T, K_R, , C_T, C_R, C_H$ were already determined using the assumptions described in section 2.1 of this report.

$$Number\ of\ load\ cycles\ for\ gear\ 4, N_{L_4} = \frac{6.6 \times 10^8}{m_{21}m_{42}} = \frac{6.6 \times 10^8}{\left(\frac{80}{34}\right)(1)} = 2.81 \times 10^8$$

$$K_L = 1.3558\, N_L^{-0.0178} = 1.3558\,(2.81 \times 10^8)^{-0.0178} = 0.9590$$

$$C_L = 1.4488\, N_L^{-0.023} = 1.4488\,(2.81 \times 10^8)^{-0.023} = 0.9262$$

Corrected Bending and Surface-contact fatigue strengths

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'} = \frac{0.9590}{(1)(1)}(65000) = 62{,}335.54\, psi$$

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc'} = \frac{(0.9262)(1)}{(1)(1)}(216000) = 200{,}059.76\, psi$$

Bending and surface-contact safety factors for gear 4

$$N_{fb} = \frac{S_{fb}}{\sigma_b} = \frac{62{,}335.54}{18{,}595.32} = 1.79$$

$$N_{fc} = \frac{S_{fc}}{\sigma_c} = \frac{200{,}059.76}{147{,}026.091} = 1.51$$

Annex D: Shafts Calculations

Preliminary Calculations

As a reference, the design process for the output and input shafts will be displayed.

First, the preliminary parameters were calculated. From the values of stress concentration factors, the fatigue concentration factors could be calculated using the notch sensitivity. Note the sample calculation using the value in bending. The same process was done for all different factors.

$K_f = 1 + q(K_t - 1)$

$K_f = 1 + 0.50(3.5 - 1)$

$K_f = 2.25$

Next, the selected material of machined AISI 1020 Steel was used to calculate the endurance strength. From Figure 7, it can be found that the uncorrected endurance strength for steels is half the ultimate strength, considering that this material's ultimate tensile strength is below 200 kpsi.

$S_e{}' = 0.5S_{ut}$

$S_e{}' = 0.5 * 65\ 000 = 32\ 500\ psi$

To calculate the corrected endurance limit, equation (A) in the Theoretical Development section is used. The adequate coefficients were also chosen based on the corresponding factors.

$$S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e{}'$$

$C_{surf} = A(S_{ut})^b$

$C_{surf} = 4.51(65000)^{-0.265}$

$C_{surf} = 0.84$

The corrected endurance limit can be calculated.

$S_e = (1)(1)(0.84)(1)(0.62)(32500) = 16926\ psi$

To justify the affirmation that $K_f = K_{fm}$, the maximum nominal stress will be compared to the ratio of yield stress over the fatigue concentration factor. The maximum nominal stress will be calculated for maximum loading of the input shaft, i.e. d_0 and will be compared with the bending fatigue concentration factor.

$$\sigma_{max,nom} = \sqrt{(2.25 * 1.5762)^2 + \frac{3}{4}(1.57 * 0)^2} + \sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}$$

$$\sigma_{max,nom} = 938.31\ psi$$

$$\frac{S_y}{K_f} = \frac{38000}{2.25} = 16889\ psi\ > \sigma_{max,nom} = 938.31\ psi$$

Hence, the assumption is valid.

Input Shaft

Torque calculations:

$$T = \frac{P}{\omega} = \frac{(60)(6600)}{(5500)\left(\frac{2\pi}{60}\right)} = 687.55\ lb * in$$

Moments and forces calculations:

$$\sum M_A = -W_{G1} * a + F_{B1} * (a + b) = 0$$

$$F_{B1} = \frac{W_{G1}(a)}{(a + b)} = \frac{(1.18215)(2)}{(2 + 1)} = 0.7881\ lb$$

Note that W_{G1} refers to the weight of the gears and F_{B1} to the reaction force of B1.

$$\sum F_y = -W_{G1} + F_{B1} + F_{B2} = 0$$

$$F_{B2} = W_{G1} - F_{B1} = 1.18215 - 0.7881 = 0.39405$$

$$M_{B1} = 0\ (from\ geometry)$$

$$M_{G1} = F_{B1} * a = (0.7881)(2) = 1.5762\ lb * in$$

$$M_{B2} = F_{B1} * a - (W_{G1} + F_{B2}) * (b) = (0.7881)(2) - (1.18215 + 0.39405) * (1)$$

$$M_{B2} = 0\ lb * in$$

Diameters Calculations

Calculating the diameters using equation (B), based on fatigue failure,

$$d_0 = \left\{ \frac{32(1.5)}{\pi} \left[\frac{\sqrt{(2.25 * 1.5762)^2 + \frac{3}{4}(1.57 * 0)^2}}{16926} + \frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}}{65000} \right] \right\}^{\frac{1}{3}}$$

$$d_0 = 0.6063609\ in$$

$$d_1 = \left\{ \frac{32(1.5)}{\pi} \left[\frac{\sqrt{(2.5 * 1.5762)^2 + \frac{3}{4}(2.7 * 0)^2}}{16926} + \frac{\sqrt{(2.5 * 0)^2 + \frac{3}{4}(2.7 * 687.55)^2}}{65000} \right] \right\}^{\frac{1}{3}}$$

$$d_1 = 0.7252399\ in$$

$$d_2 = \left\{ \frac{32(1.5)}{\pi} \left[\frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 0)^2}}{16926} + \frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}}{65000} \right] \right\}^{\frac{1}{3}}$$

$$d_2 = 0.6034446\ in$$

Now calculating the diameters using equation (H), based on yielding failure,

$$d_0 = \left\{ \frac{32(1.5)}{\pi} \left[\frac{\sqrt{(2.25 * 1.5762)^2 + \frac{3}{4}(1.57 * 0)^2}}{38000} + \frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}}{38000} \right] \right\}^{\frac{1}{3}}$$

$$d_0 = 0.7225958\ in$$

$$d_1 = \left\{ \frac{32(1.5)}{\pi} \left[\frac{\sqrt{(2.5 * 1.5762)^2 + \frac{3}{4}(2.7 * 0)^2}}{38000} + \frac{\sqrt{(2.5 * 0)^2 + \frac{3}{4}(2.7 * 687.55)^2}}{38000} \right] \right\}^{\frac{1}{3}}$$

$$d_1 = 0.8563459\ in$$

$$d_2 = \left\{ \frac{32(1.5)}{\pi} \left[\frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 0)^2}}{38000} + \frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}}{38000} \right] \right\}^{\frac{1}{3}}$$

$$d_2 = 0.7216844\ in$$

Safety factors calculations:

The final factor of safety values was hence calculated using equations (FF) and (GG). Below is a sample calculation for the diameter d_0.

$$N_f = \frac{\pi d^3}{32} \left[\frac{\sqrt{\left(K_f M_a\right)^2 + \frac{3}{4}\left(K_{fs} T_a\right)^2}}{S_e} + \frac{\sqrt{\left(K_{fm} M_m\right)^2 + \frac{3}{4}\left(K_{fsm} T_m\right)^2}}{S_{ut}} \right]^{-1}$$

$$N_f = \frac{\pi 1^3}{32} \left[\frac{\sqrt{(2.25 * 1.5762)^2 + \frac{3}{4}(1.57 * 0)^2}}{16926} + \frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}}{65000} \right]^{-1}$$

$$N_f = 6.728180883$$

$$N_y = \frac{\pi d^3}{32} \left[\frac{\sqrt{\left(K_f M_a\right)^2 + \frac{3}{4}\left(K_{fs} T_a\right)^2}}{S_y} + \frac{\sqrt{\left(K_{fm} M_m\right)^2 + \frac{3}{4}\left(K_{fsm} T_m\right)^2}}{S_y} \right]^{-1}$$

$$N_y = \frac{\pi 1^3}{32} \left[\frac{\sqrt{(2.25 * 1.5762)^2 + \frac{3}{4}(1.57 * 0)^2}}{38000} + \frac{\sqrt{(2.25 * 0)^2 + \frac{3}{4}(1.57 * 687.55)^2}}{38000} \right]^{-1}$$

$$N_y = 3.975620078$$

Output Shaft

Diameters calculations:

The same process as outlined in the input shaft calculations can be applied to calculate the moments for the output shaft. However, the diameters were calculated iteratively using an excel spreadsheet.

The diameters based on the fatigue factors of safety were calculated using equation (D) above. Diameters were selected when the factor of safety approached the desired value of 1.5.

$$1.5 = \left\{ \frac{32 * 2.25 * 177.3165882}{16926 * \pi {d_0}^3} + \frac{\sqrt{\left(\frac{4 * 2.25 * 1000}{\pi {d_0}^2}\right)^2 + 3\left(\frac{16 * 1.57 * 4537.3398}{\pi {d_0}^3}\right)^2}}{65000} \right\}^{-1}$$

$d_0 = 1.47590\ in$

$$1.5 = \left\{ \frac{32 * 2.5 * 177.3165882}{16926 * \pi {d_1}^3} + \frac{\sqrt{\left(\frac{4 * 2.5 * 1000}{\pi {d_1}^2}\right)^2 + 3\left(\frac{16 * 2.7 * 4537.3398}{\pi {d_1}^3}\right)^2}}{65000} \right\}^{-1}$$

$d_1 = 1.42474\ in$

$$1.5 = \left\{ \frac{32 * 2.25 * 868.3014219}{16926 * \pi {d_2}^3} + \frac{\sqrt{\left(\frac{4 * 2.25 * 1000}{\pi {d_2}^2}\right)^2 + 3\left(\frac{16 * 1.57 * 4537.3398}{\pi {d_2}^3}\right)^2}}{65000} \right\}^{-1}$$

$d_2 = 1.47585\ in$

$$1.5 = \left\{ \frac{32 * 2.25 * 0}{16926 * \pi {d_3}^3} + \frac{\sqrt{\left(\frac{4 * 2.25 * 1000}{\pi {d_3}^2}\right)^2 + 3\left(\frac{16 * 1.57 * 4537.3398}{\pi {d_3}^3}\right)^2}}{65000} \right\}^{-1}$$

$d_3 = 1.21871\ in$

Finally, the corrected diameters were calculated using equation (30) above.

$$1.5 = \left\{ \frac{32 * 2.25 * 177.3165882}{38000 * \pi {d_0}^3} + \frac{\sqrt{\left(\frac{4 * 2.25 * 1000}{\pi {d_0}^2}\right)^2 + 3\left(\frac{16 * 1.57 * 4537.3398}{\pi {d_0}^3}\right)^2}}{38000} \right\}^{-1}$$

$d_0 = 1.38238\ in$

$$1.5 = \left\{ \frac{32 * 2.5 * 177.3165882}{38000 * \pi {d_1}^3} + \frac{\sqrt{\left(\frac{4 * 2.5 * 1000}{\pi {d_1}^2}\right)^2 + 3\left(\frac{16 * 2.7 * 4537.3398}{\pi {d_1}^3}\right)^2}}{38000} \right\}^{-1}$$

$d_1 = 1.64390\ in$

$$1.5 = \left\{ \frac{32 * 2.25 * 868.3014219}{38000 * \pi {d_2}^3} + \frac{\sqrt{\left(\frac{4 * 2.25 * 1000}{\pi {d_2}^2}\right)^2 + 3\left(\frac{16 * 1.57 * 4537.3398}{\pi {d_2}^3}\right)^2}}{38000} \right\}^{-1}$$

$d_2 = 1.48391\ in$

$$1.5 = \left\{ \frac{32 * 2.25 * 0}{38000 * \pi {d_3}^3} + \frac{\sqrt{\left(\frac{4 * 2.25 * 1000}{\pi {d_3}^2}\right)^2 + 3\left(\frac{16 * 1.57 * 4537.3398}{\pi {d_3}^3}\right)^2}}{38000} \right\}^{-1}$$

$d_3 = 1.35380\ in$

The factors of safety for the final diameters can be calculated in the same fashion as in the sample calculations for the input shaft.

Annex E: Bearing Calculations

Sample Calculation for Combined Radial and Thrust Loading for Bearing 8

We run calculations for an SKF 33208 single row tapered roller bearing, at a location that experiences both axial and radial loading.

First, we want to verify that the bearing's rated operating speed is greater than the maximum anticipated operating speed, with a safety factor of SF=1.5:

$$\omega_{rated} > SF \times \omega_{\max anticipated}$$

$$8{,}500\ rpm > 1.5 \times 835\ rpm$$

$$8500\ rpm > \ 1{,}252.5\ rpm\checkmark$$

Second, we want to verify that the bearing's load ratings are greater than the applied loads on the bearing with a safety factor of SF=1.5:

$$F_{reaction} = F_a + F_r = 417.97\ lb + 1{,}000\ lb = 1{,}417.97\ lb$$

$$C_0 = 29{,}675\ lb > SF \times F_{reaction} = 1.5 \times 1{,}417.9\ lb = 2{,}126.85\ lb$$

$$29{,}675\ lb > \ 2{,}126.85\ lb\checkmark$$

Next, we want to verify that the bearing's estimated number of cycles is larger than the anticipated number of cycles that the bearing will experience while in operation:

$$\frac{F_a}{C_0} = \frac{1000\ lb}{29675\ lb} = 3.37 \times 10^{-2}$$

From Fig. 11-24 from Norton, we get $e = 0.22 + \frac{0.0337-0.022}{0.056-0.022} \times (0.26 - 0.22) = 0.234$

$$\frac{F_a}{V \cdot F_r} = \frac{1000\ lb}{1 \cdot 417.97\ lb} = 2.39$$

Where we set V=1, as the inner ring of the bearing rotates in our setup.

We observe that $\frac{F_a}{V \cdot F_r} = 2.34 > \ e = 3.37 \times 10^{-2}$. From Fig. 11-24 from Norton, we determine that:

X=0.56

$$Y = 1.99 + \frac{0.0337 - 0.022}{0.056 - 0.022} \times (1.71 - 1.99) = \ 1.89$$

Y=1.89

We then calculate the equivalent load:

$$P = X \cdot V \cdot F_r + Y \cdot F_a$$

$$P = 0.56 \times 1 \times 417.97\ lb + \ 1.89 \times 1000\ lb = \ 2{,}124.1\ lb$$

Given this equivalent load, we want to check that the static load rating of the bearing can withstand the equivalent load with a safety factor of SF=1.5:

$$C_0 > P \times SF$$

$$29{,}675\ lb > 2{,}124.1\ lb \times 1.5$$

$$29{,}675\ lb > 3{,}186.5\ lb$$

For an additional sanity check, we also observe that F_r = 417.97 lb and F_a = 1,000 lb. Even if we were to take the magnitude of the resulting force vector from the sum of these two components, we would still satisfy the static load rating. Once again, the main calculation is the one above (to check that C_0 > P x SF), though we want to do this following calculation as a sanity check:

$$C_0 > \ SF \times \sqrt{F_a^2 + F_r^2}$$

$$29{,}675\ lb > 1.5 \times \sqrt{(1{,}000 lb)^2 + (417.97\ lb)^2}$$

$$29{,}675\ lb > \ 1{,}500.3\ lb$$

Moving back to load cycle calculations, we then calculate the number of cycles at a L-10 lifecycle. Since we are using a roller bearing, we use an exponent of 10/3:

$$L_{10} = \left(\frac{C}{P}\right)^{\frac{10}{3}} = \left(\frac{28{,}776\ lb}{2{,}124.1\ lb}\right)^{\frac{10}{3}} = 5.93 \times 10^3\ million\ rev = 5.93 \times 10^9\ rev$$

Next, we calculate the L-1 lifecycle, as we choose a 1% roller failure rate as our design criteria from Table 11-5 from Norton:

$$L_1 = K_{R=0.99} \cdot L_{10} = 0.21 \times 5.93 \times 10^9 = 1.24 \times 10^9\ rev$$

We calculate the number of anticipated of loading cycles that the bearing needs to survive, assuming that the bearing withstands maximum loading throughout the entire flight time:

$$N_{anticipated} = 835\ rpm \times 60 \frac{min}{hr} \times 2000\ hrs = 1.002 \times 10^8\ cycles$$

We then check whether this bearing can withstand the number of anticipated load cycles. We use a safety factor of SF=1.5:

$$SF \times N_{anticipated} < \ L_1$$

$$1.5 \times 1.002 \times 10^8\ rev < 1.24 \times 10^9\ rev$$

$$1.503 \times 10^8 rev < 1.24 \times 10^9\ rev ✓$$

In summary, we were able to verify all three design criteria with a safety factor of SF=1.5:

1. The bearing's certified rotational speed is greater than the rotational speed of its gearbox shaft.
2. The bearing's static load ratings are greater than the anticipated applied loads.
3. The bearing calculated lifetime with a reliability of R=0.99 (1% bearing roller failure) is greater than the anticipated number of cycles.

□

Sample Calculation for Radial Loading for Bearing 7

We run calculations for bearing 7, which experiences radial loading. We are using the SKF NJ 2305 ECML bearing.

First, we want to verify that the bearing's rated operating speed is greater than the maximum anticipated operating speed, with a safety factor of SF=1.5:

$$\omega_{rated} > SF \times \omega_{propeller} \times gear\ ratio$$

$$12{,}000\ rpm > 1.5 \times 835\ rpm \times 2.8$$

$$12{,}000\ rpm > \ 3{,}507\ rpm\ ✓$$

Second, we want to verify that the bearing's load ratings are greater than the applied loads on the bearing with a safety factor of SF=1.5. Based on the textbook example from Norton, we calculate the magnitude of the sum of the y- and z-direction reaction loads:

$$P = \sqrt{F_y^2 + F_z^2} = \sqrt{(-780.22lb)^2 + (121.35lb)^2} = 789.6\ lb$$

We then compare the resulting applied load with the bearing's load ratings:

$$C_0 > SF \times P$$

$$12{,}364\ lb > 1.5 \times 789.6\ lb$$

$$12{,}364\ lb > \ 1{,}184.4\ lb\ ✓$$

Finally, we calculate the number of lifecycles that this bearing can withstand. We first calculate the L_{10} life of the bearing. As we are using a roller bearing, we set the exponent to 10/3:

$$L_{10} = \left(\frac{C}{P}\right)^{\frac{10}{3}} = \left(\frac{14{,}388\ lb}{789.6\ lb}\right)^{\frac{10}{3}} = 1.59 \times 10^4\ million\ revs = 1.59 \times 10^{10}\ revs$$

As we want a bearing failure rate of 1% (a reliability rate of 99%), we set the reliability factor $K_{R=0.99} = 0.21$ as defined by Table 11-5 from Norton. We get:

$$L_1 = K_{R=0.99} \times L_{10} = 0.21 \times 1.59 \times 10^{10} revs = \ 3.34 \times 10^{9}\ revs$$

We calculate the number of anticipated of loading cycles that the bearing needs to survive, assuming that the bearing withstands maximum loading throughout the entire flight time. We use an adjusted gear ratio of 2.8:

$$N_{anticipated} = 835\ rpm \times 60 \frac{min}{hr} \times 2000\ hrs \times 2.8 = 2.8056 \times 10^{8}\ cycles$$

We then check whether this bearing can withstand the number of anticipated load cycles. We use a safety factor of SF=1.5:

$$SF \times N_{anticipated} < L_1$$

$$1.5 \times 2.8056 \times 10^{8}\ rev < 3.34 \times 10^{9}\ rev$$

$$4.2084\ \times 10^{8} rev < 3.34 \times 10^{9}\ rev\checkmark$$

Just as we did for bearing 8, we were able to verify the following design criteria for bearing 7 under radial loading, with a safety factor of SF=1.5:

1. The bearing's certified rotational speed is greater than the rotational speed of its gearbox shaft.
2. The bearing's static load ratings are greater than the anticipated applied loads.
3. The bearing calculated lifetime with a reliability of R=0.99 (1% bearing roller failure) is greater than the anticipated number of cycles.

Bearings 1 to 7 are all bearings that only have a radial load, so the same above calculation can be applied to these bearings.

Full Bearing Intermediary Calculations

B1 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_B1	0.788	lb		
Real world RPM	5500	rpm		From problem statement
C	6407	lb	SKF N204 ECP	
C_0	4946	lb	SKF N204 ECP	
Max RPM	19000	rpm	SKF N204 ECP	Limiting speed
Desired RPM	6.600E+08	revs		
K_R	0.21		1% failure rate	

B1 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	0.788	lbs
L_10	1.08E+13	millions of revs
L_10	1.08E+19	revs
L_P	2.27E+18	revs

B2 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_B2	0.394	lb		
Real world RPM	5500	rpm		From problem statement
C	6407	lb	SKF N204 ECP	
C_0	4946	lb	SKF N204 ECP	
Max RPM	19000	rpm	SKF N204 ECP	Limiting speed
Desired RPM	6.600E+08	revs		
K_R	0.21		1% failure rate	

B2 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	0.394	lbs
L_10	1.09E+14	millions of revs
L_10	1.09E+20	revs
L_P	2.29E+19	revs

B3 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_y	34.83342658	lb		
F_z	121.3548595	lb		
Real world RPM	2338	rpm		
C	3192	lb	SKF N205 ECP	
C_0	2967	lb	SKF N205 ECP	
Max RPM	18000	rpm	SKF N205 ECP	Limiting speed
Desired L_10	2.806E+08	revs		
K_R	0.21		1% failure rate	

B3 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	126.2551762	lbs
L_10	4.74E+04	millions of revs
L_10	4.74E+10	revs
L_P	9.96E+09	revs

B4 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_y	-21.95356571	lb		
F_z	116.6270411	lb		
Real world RPM	835	rpm	Propeller max RPM	Problem statement
C	3192	lb	SKF N205 ECP	
C_0	2967	lb	SKF N205 ECP	
Max RPM	18000	rpm	SKF N205 ECP	Limiting speed
Desired L_10	2.806E+08	revs		
K_R	0.21		1% failure rate	

B4 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	118.6752955	lbs
L_10	5.83E+04	millions of revs
L_10	5.83E+10	revs
L_P	1.22E+10	revs

B5 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_y	783.9571303	lb		
F_z	121.3548595	lb		
Real world RPM	835	rpm		
C	14388	lb	NJ 2305 ECML	
C_0	12364	lb	NJ 2305 ECML	
Max RPM	22000	rpm	NJ 2305 ECML	Limiting speed
Desired L_10	2.806E+08	revs		
K_R	0.21		1% failure rate	

B5 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	793.2942607	lbs
L_10	1.57E+04	millions of revs
L_10	1.57E+10	revs
L_P	3.29E+09	revs

B6 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_B6	177.317	lb		
Real world RPM	835	rpm	Propeller max RPM	Problem statement
C	6407	lb	NU 1008 ML	
C_0	5845	lb	NU 1008 ML	
Max RPM	18000	rpm	NU 1008 ML	Limiting speed
Desired RPM	1.002E+08	revs		
K_R	0.21		1% failure rate	

B6 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	177.317	lbs
L_10	1.56E+05	millions of revs
L_10	1.56E+11	revs
L_P	3.28E+10	revs

B7 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_y	-780.2202912	lb		
F_z	121.3548595	lb		
Real world RPM	2338	rpm	Propeller max RPM	
C	14388	lb	NJ 2305 ECML	
C_0	12364	lb	NJ 2305 ECML	
Max RPM	22000	rpm	NJ 2305 ECML	Limiting speed
Desired L_10	2.806E+08	revs		
K_R	0.21		1% failure rate	

B7 Bearing: Results		
Term	Value	Unit
K_R	0.21	N/A
P	789.6016114	lbs
L_10	1.59E+04	millions of revs
L_10	1.59E+10	revs
L_P	3.34E+09	revs

B8 Bearing: Given Data				
Term	Value	Unit	Notes	Source
F_a	1000	lb		
F_r	417.9727993	lb		
Real world RPM	835	rpm	Propeller max RPM	
C	28776	lb	33208	
C_0	29675	lb	33208	
Max RPM	8500	rpm	33208	Limiting speed
V	1			
e	0.0337			
X	0.56			
Y	1.89			
Desired L_10	1.002E+08	revs		
K_R	0.21		1% failure rate	

B8 Bearing: Results		
Term	Value	Unit
F_a/C_0	0.033698399	
F_a/(V*F_r)	2.392500186	
P	2124.064768	lb
L_10	5927.554629	millions of revs
L_10	5.9276E+09	revs
L_P	1.2448E+09	revs

Annex F: Key Calculations

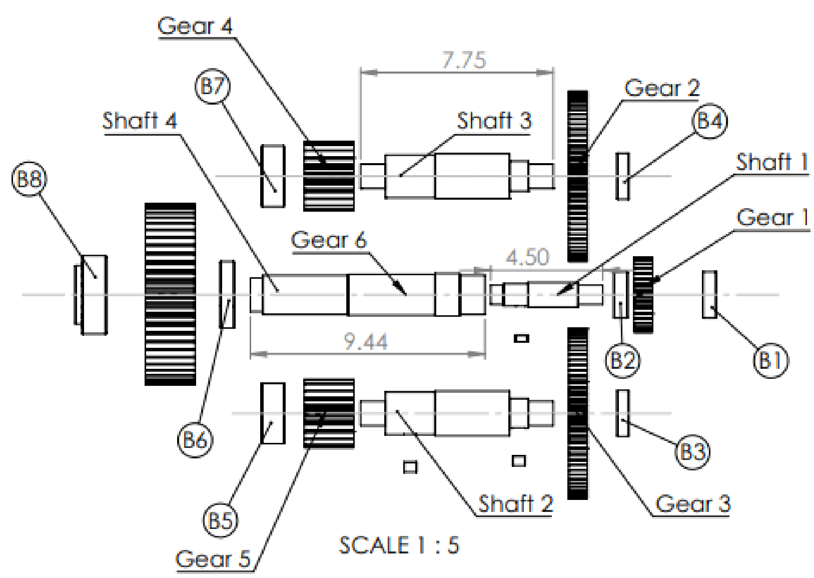
Key 1:

$$\tau = \frac{1527lb}{0.5in * 0.25in} = 12223.1\ psi, \qquad \sigma' = \sqrt{3}\tau = 21171\ psi, \qquad N_{shear} = \frac{53000\ psi}{\sigma'} = 2.50$$

$$\sigma_{bearing} = \frac{1527\ lb}{\frac{1}{2} * 0.5in * 0.25in} = 24446\ psi, \qquad N_{bearing} = \frac{44000\ psi}{\sigma_{bearing}} = 1.80$$

6. Drawings

Drawing 6.1: Exploded Gearbox



GEARS	Face Width [in]	Pitch Diameter [in]	Bore Diameter [in]
Gear 1	0.75	2.83	0.9
Gear 2	0.75	6.67	1.2
Gear 3	0.75	6.67	1.2
Gear 4	2	2.5	1.71
Gear 5	2	2.5	1.71
Gear 6	2	7	1.65

SHAFTS	Shaft Length
Shaft 1	4.50 in
Shaft 2	7.75 in
Shaft 3	7.75 in
Shaft 4	9.44 in

Bearing Label	Bearing Name
B1, B2	SKF N204 ECP
B3, B4	SKF NU1005
B5, B7	SKF NJ 2305 ECML
B6	SKF NU 1008 ML
B8	SKF 33208

UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN INCHES SURFACE FINISH: TOLERANCES: LINEAR: ANGULAR:

FINISH: Machine Finish

DEBURR AND BREAK SHARP EDGES

DO NOT SCALE DRAWING | REVISION

NAME | SIGNATURE | DATE

DRAWN | CHK'D | APPV'D | MFG | Q.A

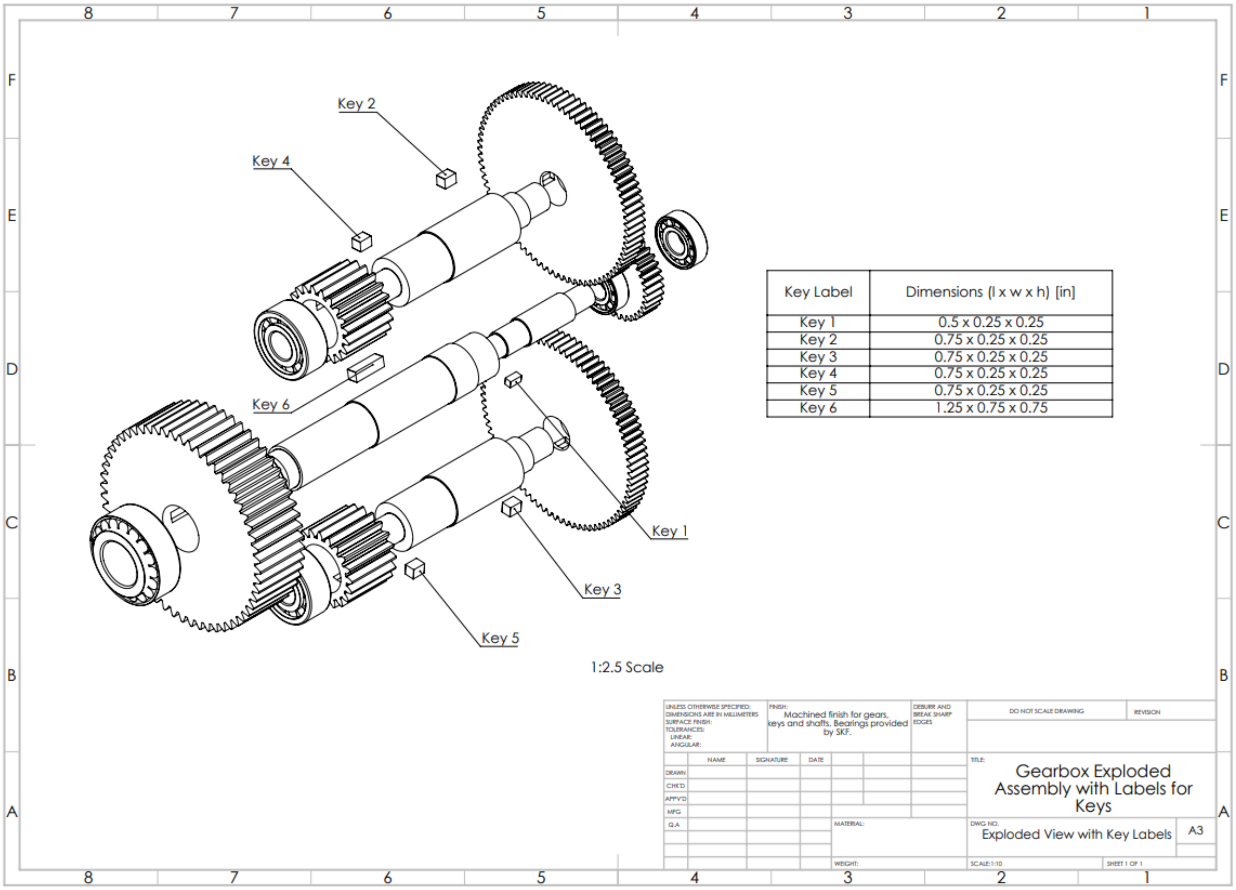
MATERIAL:

TITLE: Exploded Gearbox

DWG NO. Labeled Assembly Exploded View | A4

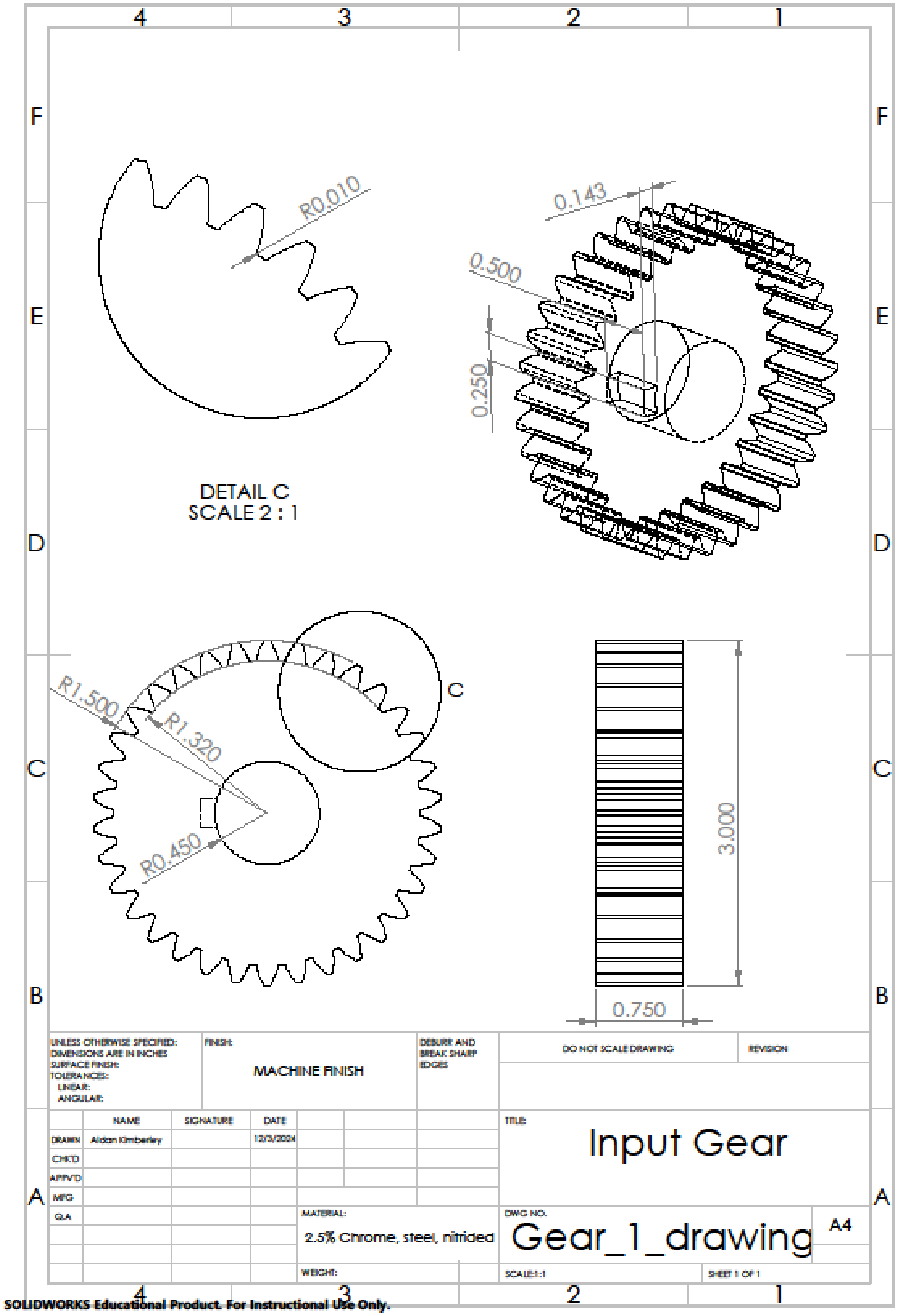
WEIGHT: | SCALE:1:10 | SHEET 1 OF 1

Drawing 6.2: Gearbox Exploded Assembly with Labels for Keys



Key Label	Dimensions (l x w x h) [in]
Key 1	0.5 x 0.25 x 0.25
Key 2	0.75 x 0.25 x 0.25
Key 3	0.75 x 0.25 x 0.25
Key 4	0.75 x 0.25 x 0.25
Key 5	0.75 x 0.25 x 0.25
Key 6	1.25 x 0.75 x 0.75

Drawing 6.3: Input Gear



Drawing 6.4: Output Shaft